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330:501 Systems Analysis

Problem Set 6 SOLUTIONS

S & B: Ch.[4], prob. 1:
We have found the transition matrix in P3-1:

$$\phi(t,\tau) = \begin{bmatrix} t/\tau & 0 \\ 0 & t/\tau \end{bmatrix}$$

For $t > \tau$, $\phi(t,\tau)$ is not bounded which implies that this system is unstable.

S & B: Ch.[4], prob. 2:
We have found the transition matrix in P2-4 (refer to the previous solution). $\phi(t,t_0)$ is not bounded for $t > t_0$, so the system is not stable.

S & B: Ch.[4], prob. 4:
The transition matrix has been found in P3-7 as

$$\phi(t,t_0) = \begin{bmatrix} 1 & 1/6 - 1/6e^{-6(t-t_0)} \\ 0 & e^{-6(t-t_0)} \end{bmatrix}$$

which is bounded for $t \geq t_0$. Therefore the system is stable, but not asymtotically stable (has an eigenvalue of zero).

S & B: Ch.[4], prob. 6:
For this discrete system, we have the characteristic function $(2-\lambda)^2 = 0$. The solutions are $\lambda_1 = \lambda_2 = 2$. Since the magnitudes of the eigenvalues are greater than 1, the system is unstable.

S & B: Ch.[4], prob. 8:
We can form the Routh array for this system,

$$\begin{array}{cccc} s^3 & 1 & 3 & 0 \\ s^2 & 2 & 5 & 0 \\ s & 1/2 & 0 & \\ s^0 & 5 & & \end{array}$$

There is no change of sign in the first column of the array. Therefore this is a stable system. We can also form the Hurwitz array as

$$\begin{bmatrix} 2 & 5 & 0 \\ 1 & 3 & 0 \\ 0 & 2 & 5 \end{bmatrix}$$

and find all the appropriate determinants (2, 1 and 5) greater than zero.

S & B: Ch.[4], prob. 12:
We have $V(x) = (x - \bar{x})^T(x - \bar{x})$. This function satisfies the first two conditions of the Lyapunov function in that it is continuous and has a unique minimum at the stationary point $\bar{x}$. We check the last condition by differentiating the $V(x)$ with respect to time to obtain.

$$\dot{V}(x) = \dot{x}^T(x - \bar{x}) + (x - \bar{x})^T\dot{x} = 2\dot{x}^T(x - \bar{x})$$

We require

$$\dot{x}^T(x - \bar{x}) = f^T(x)(x - \bar{x}) \leq 0$$

Thus, the latter part of the equation (in $f()$) specifies the condition on $f(x)$ $\forall x \in \Omega$ for $V(x)$ to be a Lyapunov function.

NOTE: This might be considered as a generalized form of *negative feedback.* However, note that not all components of $f(x)$ need have inverted signs from the corresponding components of $(x - \bar{x})$. Neat, huh?!