Rutgers University
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330:501 Systems Analysis

Problem Set 5

1. **S & B: Chapter 3, Problem 13**
2. **S & B: Chapter 3, Problem 15**
3. **S & B: Chapter 3, Problem 17**
4. **S & B: Chapter 3, Problem 20**
5. **S & B: Chapter 3, Problem 21**
6. Consider the diagram of FIGURE 1 where N identical cylindrical buckets of cross sectional area $A = 1$ meter^2 are connected by a set of pumps. Each pump is labeled with a flow direction and a symbol which indicates the power of the pump. Specifically, if $\alpha \geq 0$ is the pump power then it generates a volume flow of $Q(t) = \alpha x(t)$ meter^3/sec from input to output where α is a constant and $x(t)$ is the height of the liquid at the pump inlet.

   The liquid is incompressible and does not evaporate. At time $t = 0$ the heights of liquid are $x_1 = 1$ meter and $x_i = 0$ for all other buckets. Assume that the amount of liquid contained in the idealized pumps and their associated tubing is essentially zero at all times (even when liquid is passing through them); i.e., all liquid in the system is contained in the buckets and nowhere else.

   (a) Write down the differential equations which relate the heights $\{x_i\}$ of fluid in the buckets as a function of time.

   (b) Render the system in statespace form $\dot{\mathbf{w}} = \mathbf{A}\mathbf{w}$ where

   $$\mathbf{w} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$$

   and $\mathbf{A}$ is an $N \times N$ matrix.

   (c) Derive a minimal state representation of the system using state vector

   $$\mathbf{v} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N-1} \end{bmatrix}$$

   State all assumptions.

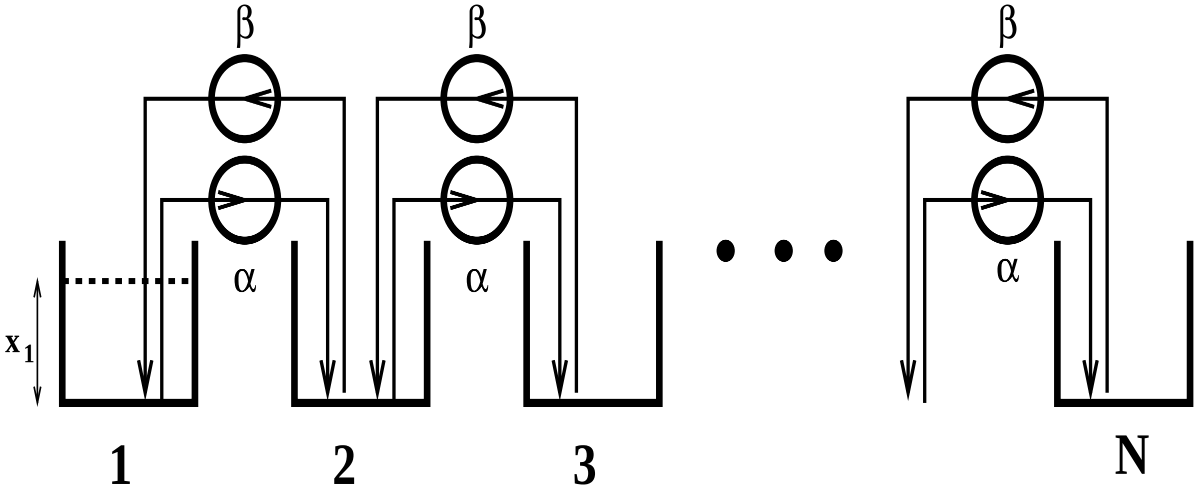

Figure 1: Diagram for bucket problem.

(d) Suppose $x_1(0) = 1$ meter and $x_i(0) = 0$, $i = 2, 3, ...N$. Further suppose that $\beta = 0$ and $\alpha = 1$. Derive an expression for $x_1(t)$ and $x_N(t)$ for $t \geq 0$. Does a fixed point exist? If so, what is it?