Rutgers University
The State University Of New Jersey
College of Engineering
Department of Electrical and Computer Engineering

330:501 | Systems Analysis | Fall 1995

POP QUIZ

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This "examlette" will hopefully help you prepare for the final. I expect it to take about 30 to 45 minutes to do. Then while it's still fresh in our minds we'll all go over it. So relax and let's find out what is confusing and what's well-known.

1. Consider the following systems and determine the number of fixed points, the number of limit points and whether the implied mapping is a contraction over the space indicated for time $t \geq t_0$. If there is not enough information given then say so.

   (a) A frog who never moves is placed somewhere on a frog racecourse at time $t = t_0$.

   (b) A marble is tossed with random trajectory into a slightly tilted muffin tin at time $t = t_0$. If you don't know what a muffin tin is, move on and don't worry.

   (c) A child asleep on a swing (assume seat can hold the child!).

   (d) The system
   $$\dot{x} = Ax$$
   where A is a square $n \times n$ matrix of zeros.

   (e) Same as above only the matrix $A = I$.

   (f) Same as above only the matrix $A = -I$.

   (g) The system
   $$\dot{x} = Ax$$
   where A has all eigenvalues with negative real parts except for one which has value zero; i.e., the matrix A has rank $n - 1$.

2. **Basic systems problems:**

   (a) Find the transition matrix for the system
   $$\dot{x} = \begin{bmatrix} -t & 0 \\ 0 & -\cos t \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$
   Does this system have a transfer function?

   (b) Find the transition matrix for the system
   $$\dot{x} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$
   Does this system have a transfer function?

3. **Stability, Observability, Controllability:**

(a) Which of the systems in the previous problem are stable, asymptotically stable, globally asymptotically stable, BIBO stable?

(b) Which of the systems are controllable?

(c) Which of the systems are observable? Assume that $y = [1, 0]x$.

4. FINAL QUESTION!

You are given the system

$$\dot{x} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & -4 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

(a) Assume $x(0) = 0$ and show how to devise a control $u(t)$ (if possible) which makes the output $y(t) = 1 - e^{-t}$ on $t \in [0, 1]$.

(b) Now suppose the output is given by $y = [1, 0, 1]x$. Given $u(t)$ and $y(t)$ on $[t_0, t_1]$ can $x(t_0)$ be uniquely determined?

(c) Given $u(t) = 0$ and output $y(t) = e^{3t} \sin 2t$ on $t \in [0, 1]$, can $x(0)$ be determined? Please think CAREFULLY.

5. WASN'T THAT FUN!?!?!?!?!?