School of Engineering
Department of Electrical and Computer Engineering

332:421 **Wireless Communications Systems** **Fall 2010**

Take Home Examination

There are three problems on this examination. Two require programming. The exam is due (electronically: pdf preferred, but word/etc or a scan of handwritten work ok) Monday November 22, 2010 at 11:59PM. You may work in groups of up to five people. If you do work in groups, you must carefully do credit assignment (who did what and how much). You will also be asked privately to grade fellow group members. NOTE: This exam is similar to but not identical with last year's takehome.
GOOD LUCK!

1. *(50 points)* **PN Sequences:** We are building an in-class CDMA system. We will use a 31-bit maximal-length pseudo-random noise (PN) sequence generator with taps at 31 and 28 as depicted in FIGURE 1. Bits 31 and 28 are exclusive-OR'ed and the result fed back into the shift register. Bit 31 is defined as the shift register output. The 31-bit binary number $\mathbf{x}_0$ is defined as the initial shift register (SR) state.

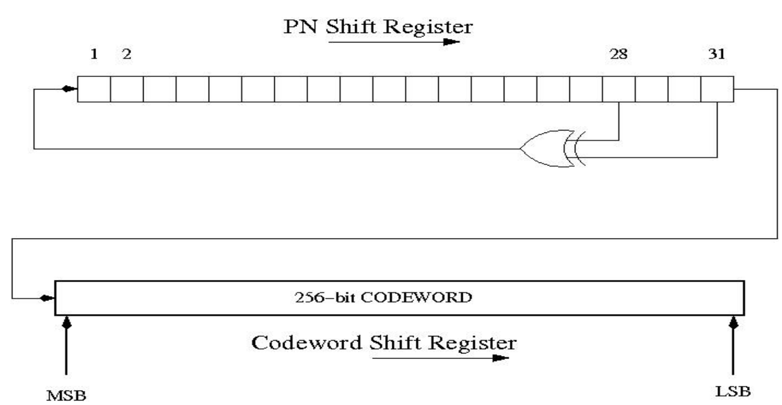

Figure 1: 31-bit Maximal Length PN Shift Register. Second part of figure (Codeword) is for problem 2.

   (a) *(10 points)* What is SR output if $\mathbf{x}_0 = \mathbf{0}$.

   (b) *(10 points)* What is the state of the SR after 31 shifts if $\mathbf{x}_0 = 7ffffff$ (hex)?

   (c) *(10 points)* Is this shift register sequence periodic for non-zero $\mathbf{x}_0$? What is its period?

   (d) *(20 points)* Please verify your periodicity result numerically (Matlab, C, whatever programming language you're most comfortable with) so that the world believes you.

2. *(150 points)* **Classroom CDMA:** We now incorporate the PN sequence generator above into a CDMA system. As you already know, CDMA system employ user codewords (signatures) to transmit information. During bit interval k, user m has codeword $\mathbf{s}_{m_k}$ composed of $\pm$1s. User m sends bit $b_{m_k} = \pm 1$ by broadcasting $b_{m_k}\mathbf{s}_{m_k}$. Notice that unlike in class, the codeword here changes with each bit interval. The receiver receives

$$\mathbf{r}_k = \sum_{m=1}^{M} a_{m_k} b_{m_k} \mathbf{s}_{m_k} \tag{1}$$

where a_{m_k} is the amplitude with which the signal is received. We have previously assumed $a_{m_k} = 1$ for all users at all times, but clearly this was a pedagocial simplification. Users at different distances from the receiver will have their signals received with different powers. Also, users move around and the environment changes (a truck passes by, for instance) which leads to time-dependence.

The receiver then decodes each user's bit in interval k by using a matched filter

$$\hat{b}_{m_k} = \begin{cases} 1 & \mathbf{r}_k^\top \mathbf{s}_{m_k} > 0 \\ -1 & \mathbf{r}_k^\top \mathbf{s}_{m_k} \le 0 \end{cases}$$

In this problem, I will be the base station and will compose messages to each of you which I will assemble and broadcast to the class as in equation (1). Your seed will be the 31-bit binary representation (truncation in some cases) of your 9 digit student ID. This 31-bit number will be the initial "seed" $\mathbf{S}_m$ for your PN-sequence generator: the least significant bit (LSB) of your number will be bit 1 in the shift register while the most significant bit (MSB) will be the 31^{st}.

For coding of messages, $1 \to 1$ and $0 \to -1$ and each successive 256-bit 'slice" of the resultant sequence will be your successive codewords. That is, using seed S_m, the first 256 bits (ordered LSB to MSB) output by the shift register will comprise $\mathbf{s}_{m_1}$, the next $\mathbf{s}_{m_2}$ and so on. Finally, your messages will be coded in 7-bit ASCII format (MSB first, left to right). You can find ASCII tables on the Web.

(a) *(20 points)* There are 19 people (left :)) in the class and an additional message which points to the bonus problem. What is the expected probability of bit error for each person. You may assume that the PN sequence is actually truly random and that users transmit equally likely ± 1 i.i.d.

(b) *(30 points)* Please derive an expression for user m's SNR which includes the received amplitudes a_{n_k}. You may assume that $a_{n_k} = a_{n_\ell}$ (i.e., no time-dependence on received signal amplitude). Which user has the lowest probability of error?

(c) *(100 points)* Please write a program that will take the sequence of digits from

http://www.winlab.rutgers.edu/~crose/421_html/cdma256stream.txt

and decode your personal message. You might wish to start out with the test message (which is your name) to check your software before attempting to decode the larger message. The test stream can be found at

http://www.winlab.rutgers.edu/~crose/421_html/cdma256teststream.txt.

NOTE: Your message will contain personal information like grades and what I currently think of your strengths and weaknesses and other teacherly things, so you might want to decode these in private if you work in a group.

3. *(100 points)* **Simple Multiuser Detection:** Suppose we have a system with two users having codewords

$$\mathbf{s}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

and

$$\mathbf{s}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

   (a) *(20 points)* Sketch these two signals in signal space.

   (b) *(20 points)* If each user transmits $b_m = \pm 1$ with equal probability and the receiver sees

$$\mathbf{r} = b_1 \mathbf{s}_1 + b_2 \mathbf{s}_2$$

   what is the probability of error for each user. Carefully state your assumptions.

   (c) *(30 points)* Now, suppose that instead of trying to transmit exactly one bit per transmission, the user 1 uses two successive codewords $[\mathbf{s}_1, \mathbf{s}_1]$ when it intends to transmit a 1 and $[-\mathbf{s}_1, -\mathbf{s}_1]$ when it intends -1. Likewise, suppose user 2 transmits $[\mathbf{s}_2, -\mathbf{s}_2]$ and $[-\mathbf{s}_2, \mathbf{s}_2]$ for 1 and -1 respectively. That is, each user reduces its bit rate by a factor of two to achieve some error performance improvement. What is the probability of error for this new scheme?

   (d) *(30 points)* Now, suppose the user signals are received at different amplitudes

$$\mathbf{r} = b_1 \mathbf{s}_1 + 2 b_2 \mathbf{s}_2$$

   What is the probability of error of this system?

4. *(100 points)* **BONUS:** To access the "bonus problem", you'll have to successfully write your CDMA routine to extract it from the cdma stream provided on the course web page. The seed for the bonus question is: **710746980**.