RUTGERS

School of Engineering
Department of Electrical and Computer Engineering

332:421 **Wireless Communications Systems** **Fall 2008**

Take Home Examination

Work alone. GOOD LUCK!

1. *(50 points)* **Drunken Channel:** A binary communications system is based on two signals:

   - $s_0(t) = 0$
   - $s_1(t) = \cos(2\pi t + \theta)$ where each time $s_1(t)$ is used, θ is a(n independent) uniform random variable on $[0, 2\pi]$

   The signals are used equiprobably. The received signal is $r(t) = s_i(t) + w(t)$ where $w(t)$ is a zero mean white gaussian noise process with spectral height N_0.

   (a) *(10 points)* Please derive a signal space for these signals. Remember, the phase term θ is a random variable which changes each time $s_1(t)$ is sent.

   (b) *(20 points)* Design a correlator receiver for these signals.

   (c) *(20 points)* Derive a rule which uses the output of your receiver to provide a minimum error probability decision. Sketch the decision region boundaries in your signal space. and provide an expression for the probability that signal $s_0(t)$ is mistaken for $s_1(t)$.

2. *(50 points)* **Another Signal Basis:** Consider the set of signals on $[0, 1]$,

$$\phi_k(t) = u(t) + 2\sum_{m=1}^{2^k} (-1)^m u(t - \frac{m}{2^k})$$

   and

$$\psi_k(t) = \phi_{k+1}(t + \frac{1}{2^{k+2}})$$

   $k = 0, 1, 2, \ldots.$

   (a) *(10 points)* Sketch out $\phi_k(t)$ and $\psi_k(t)$ for $k = 0, 1, 2$.

   (b) *(10 points)* Do $\phi_k(t)$ and $\psi_k(t)$ form an orthonormal set of functions.

   (c) *(10 points)* Calculate the projection of the signal $f(t) = \sqrt{3}t$ onto the set to obtain $\{a_k\}$ and $\{b_k\}$ the projections onto the $\phi_k(t)$ and the $\psi_k(t)$ respectively. Do this analytically.

   (d) *(10 points)* Write a program to compute

$$g(t) = \sum_{k=0}^{N} a_k \phi_k(t) + \sum_{k=0}^{N} b_k \psi_k(t)$$

   and then plot your result for $N = 1$, $N = 4$ and $N = 10$.

(e) *(10 points)* Express

$$\int_0^1 [g(t) - f(t)]^2 dt$$

analytically as a function of the $\{a_k\}$ and $\{b_k\}$.

3. *(50 points)* **Non-White Noise:** A two-dimensional signal space has a single basis functions $\phi(t)$ Information is coded for transmission over a channel as

$$\begin{aligned} r_1(t) &= c\phi(t) + w_1(t) \\ r_2(t) &= c\phi(t) + w_2(t) \end{aligned}$$

where $c \in [0,1]$ is a real number and is the information symbol and the $w_i(t)$ are both zero mean stationary white jointly-Gaussian noise processes, each with spectral height 1. However, $w_1(t)$ and $w_2(t)$ are NOT independent and have

$$E\left[w_1(t)w_2(t+\tau)\right] = -\delta(\tau)$$

(a) *(30 points)* Please design an optimal receiver for this transmission scheme.

(b) *(10 points)* What is the probability of error of your receiver?

(c) *(10 points)* How many bits of information can you reliably send per channel use on average using your system?

4. *(50 points)*

Cora in Love: Cora the communications engineer has fallen hard for her colleague, Dr. Love and desperately wants him to ask her out. So, Cora contacts Dr. Emery Brown at Harvard University who has developed new technology which allows remote stimulation of brain cells. Cora plans to use Emery's invention to stimulate Dr. Love's center of affection when she's nearby, hoping he'll make a happy association between feeling affectionate and Cora's proximity.

Unfortunately, but not surprisingly, the center of affection is comingled with the center of fear. In people predisposed to paranoia, fear often elicits a violent response. And yes, you guessed it, Dr. Love has paranoid tendencies.

However, by varying two parameters, α and ϕ, Cora can control the joint PDF energy E_α which reaches the affection center and E_ϕ which reaches the fear center.

If $E_\alpha > E_\phi$, an affection response is elicited from Dr. Love. Likewise, if $E_\alpha < E_\phi$, Dr. Love strangles Cora. If $E_\alpha = E_\phi$ then affection balances fear for a net null response. Finally, if either E_α or E_ϕ exceed some threshold E_{danger}, then brain tissue is destroyed and Dr. Love becomes Dr. Lettuce – i.e., a vegetable. So, your job is to help Cora find an appropriate signal point (α, ϕ).

Owing to the complex physical interactions, the PDF on E_α and E_ϕ is a completely non-intuitive

$$f_{E_\alpha, E_\phi}(E_a, E_f) = \frac{3}{4} e^{-\left|(E_a - \alpha) - (E_f - \phi)\right|} e^{-\frac{(E_a - \alpha) + (E_f - \phi)}{2}} u(E_f - \phi) u(E_a - \alpha)$$

where $u()$ is the unit step function.

(a) *(10 points)* Verify that $f_{E_\alpha,E_\phi}(E_a,E_f)$ is indeed a PDF.

HINT: Consider two separate regions of integration and first consider the case $\alpha = \phi = 0$.

(b) *(10 points)* What is the probability that Dr. Love is vegetized in terms of E_{danger} and (α,ϕ)?

(c) *(10 points)* **Love me or die:** Suppose Cora's only desire is to maximize the probably that Dr. Love likes her (and she doesn't care whether he turns into a vegetable or not). What signal point (α,ϕ) should she choose?

(d) *(20 points)* Now, suppose Cora actually cares for Dr. Love and wishes to maximize the probability that Dr. Love likes her subject to a maximum vegetation probability criterion P_v. That this, she wishes to keep both $\text{Prob}(E_\alpha > E_{\text{danger}})$ and $\text{Prob}(E_\phi > E_{\text{danger}})$ smaller than P_v. What (α,ϕ) signal point should she choose and what is the resultant probability of Dr. Love liking her?

Evaluate your result for $E_{\text{danger}} = 10$.

HINT: Look at the "decision regions" implied by the problem and integrate appropriately.

(e) *(extra credit)* What does this problem say about the role of fear in love?

Problem 4 is obviously **completely made up** – except for the part of about Emery Brown at Harvard/MIT who *has* shown how to measure brain states (a window on the mind!) using multi-electrode recordings. Though I haven't spoken to him recently, I don't think he knows how to *effect* given brain states — at least not yet.