School of Engineering
Department of Electrical and Computer Engineering

332:421 **Wireless Communications Systems** **Fall 2010**

Quiz I

Write all answers on the printed sheet

1. *(60 points)* **Cora and the Alien Invasion:** Cora the Communications Engineer has been hired by NASA to investigate the possibility of an imminent alien invasion through measurements taken by the Voyager deep space probe as it pierces the heliosphere (extended solar neighborhood). Through top secret research, NASA has determined that the signal level S follows the following distributions:

$$f_{S|H_0}(s|H_0) = \lambda^2 s e^{-\lambda s}$$

and

$$f_{S|H_1}(s|H_1) = \lambda s e^{-\frac{1}{2}\lambda s^2}$$

where H_1 means the aliens are planning an invasion, and H_0 not. In both cases, $s \geq 0$. Your job is to help Cora design a decision box which takes the measurement S and produces a decision about whether the aliens are invading or not and does so with minimum probability of error.

   (a) *(20 points)* Please carefully sketch the two conditional distributions for $\lambda = 1$.

(b) *(20 points)* If the aliens are planning an invasion with probability p, please provide an appropriate likelihood ratio for the decision regions associated with H_0 and H_1.

(c) *(20 points)* Please determine analytic expressions for decision regions for $p = 0.5$. What does your region reduce to if $\lambda = 1$?

2. *(40 points)* **Signal Space:** A two-dimensional signal space uses basis functions $\phi_1(t) = 1$ and $\phi_2(t) = \sqrt{2}\cos 2\pi t$ on an interval $(0, 1)$.

   (a) *(10 points)* Please provide a signal space vector representation $\mathbf{s}_k$ for each of the functions $s_1(t) = \cos^2 \pi t$ and $s_2(t) = \sin^2 \pi t$ so that

   $$s_k(t) = \mathbf{s}_k^\top \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix} = s_{k1}\phi_1(t) + s_{k2}\phi_2(t)$$

   What is the energy in each signal? What is the distance between these two points in signal space?

(b) *(10 points)* On an interval $(0, 1)$ one of these signals is sent with equal probability. Zero mean white Gaussian noise of spectral height N_0, $w(t)$, is added to the transmission and a minimum probability of error receiver is used to decode whether signal 1 or signal 2 was sent. Under this scenario, the probability of error is some number P_e

Now, suppose we change the signal design and keep $s_1(t)$ as is but change $s_2(t) = -s_1(t)$. Does the total signal energy change? Does the probability of error go up or down? Why?

(c) *(20 points)* Plot the two pairs of signal points in the same signal space coordinate frame and comment on binary signal design (where you have only two possible signals) under signal energy constraints in a multidimensional signal space. You may guess (with verbal justification) but to receive full credit you must PROVE your assertion.

3. *(50 points)* **Equalization** A communication channel has impulse response

$$h_k = (-a)^k$$

for $0 < a < 1$, $k = 0, 1, ...$ and is zero otherwise.

(a) *(25 points)* What two tap equalizer c_0, c_1 corrects the distorion introduced by h_k?

(b) *(10 points)* You are asked to build an adaptive equalizer for this system. Derive an expression for $\sum_{n=0}^{N} e_n^2$ where e_n is the difference between $\hat{x}_n$ the output of your two-tap equalizer filter and x_n.

(c) *(10 points)* Derive an explicit tap update equation for your adaptive equalizer using step size Δ.

(d) *(5 points)* You are provided with a training sequence

$$x_n = (-1)^n$$

$n = 0, 1, 2, 3$. If $a = 1/2$, the initial tap weights are $c_0 = c_1 = 0$ and the step size is $\Delta = \frac{1}{10}$ what are the updated tap weights after you perform one iteration of the update procedure.