School of Engineering
Department of Electrical and Computer Engineering

332:421 **Wireless Communications Systems** **Fall 2009**

Take Home Examination

There are three problems on this examination. Two require programming. The exam is due in class on October 19, 2009. You may work in groups of up to five people. If you do, you must carefully do credit assignment (who did what and how much). You will also be asked to grade fellow group members.
GOOD LUCK!

1. *(50 points)* **PN Sequences:** You are to build an in-class CDMA system. We will use a 20-bit maximal-length pseudo-random noise (PN) sequence generator with taps at 20 and 17 as depicted in FIGURE 1. Bits 20 and 17 are exclusive-OR'ed and the result fed back into the shift register. Bit 20 is defined as the shift register output. The 20-bit binary number $\mathbf{x}_0$ is defined as the initial shift register (SR) state.

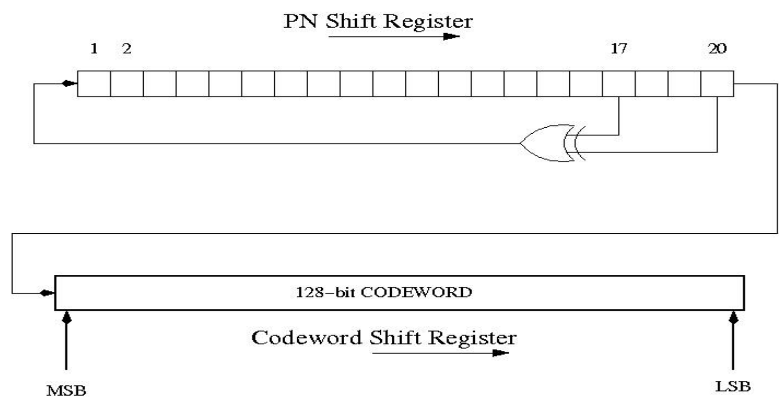

Figure 1: 20-bit Maximal Length PN Shift Register. Second part of figure (Codeword) is for problem 2.

(a) *(10 points)* What is SR output if $\mathbf{x}_0 = \mathbf{0}$.

(b) *(10 points)* What is the state of the SR after 20 shifts if $\mathbf{x}_0 = 00001$ (hex)?

(c) *(10 points)* Is this shift register sequence periodic for non-zero $\mathbf{x}_0$? What is its period?

(d) *(20 points)* Please verify your periodicity result numerically (Matlab, C, whatever programming language you're most comfortable with) so that the world believes you.

2. *(150 points)* **Classroom CDMA:** We now incorporate the PN sequence generator above into a CDMA system. As you already know, CDMA system employ user codewords (signatures) to transmit information. During bit interval k, user m has codeword $\mathbf{s}_{m_k}$ composed of ± 1s. User m sends bit $b_{m_k} = \pm 1$ by broadcasting $b_{m_k}\mathbf{s}_{m_k}$. Notice that unlike in class, the codeword here changes with each bit interval. The receiver receives

$$\mathbf{r}_k = \sum_{m=1}^{M} a_{m_k} b_{m_k} \mathbf{s}_{m_k}$$

where a_{m_k} is the amplitude with which the signal is received. We have previously assumed $a_{m_k} = 1$ for all users at all times, but clearly this was a pedagocial simplification. Users at different distances from the receiver will have their signals received with different powers. Also, users move around and the environment changes (a truck passes by, for instance) which leads to time-dependence.

The receiver then decodes each user's bit in interval k by using a matched filter

$$\hat{b}_{m_k} = \begin{cases} 1 & \mathbf{r}_k^\top \mathbf{s}_{m_k} > 0 \\ -1 & \mathbf{r}_k^\top \mathbf{s}_{m_k} \leq 0 \end{cases}$$

In this problem, I will be the base station and will compose messages to each of you which I will assemble and broadcast to the class. Your codewords will be based on the binary representation of the 2^{nd}, 3^{rd} and last 4 digits of your student ID (6 decimal digits total). This 20-bit number will be the initial "seed" $\mathbf{S}_m$ for your PN-sequence generator: the least significant bit (LSB) of your number will be bit 1 in the shift register while the most significant bit (MSB) will be the 20^{th}.

$1 \to 1$ and $0 \to -1$, each successive 128 "slice" of the resultant sequence will be your successive codewords. That is, using seed S_m, the first 128 bits (ordered LSB to MSB) output by the shift register will comprise $\mathbf{s}_{m_1}$, the next $\mathbf{s}_{m_2}$ and so on. Finally, your messages will be coded in 7-bit ASCII format (MSB first, left to right). You can find ASCII tables on the Web.

(a) *(20 points)* There are 20 people in the class. What is the probability of bit error for each person. You may assume that the PN sequence is actually truly random and that users transmit equally likely ± 1 i.i.d.

(b) *(30 points)* Please derive an expression for user m's SNR which includes the received amplitudes a_{n_k}. You may assume that $a_{n_k} = a_{n_\ell}$ (i.e., no time-dependence on received signal amplitude). Which user has the lowest probability of error?

(c) *(100 points)* Please write a program that will take the sequence of digits I supply and decode your personal message.

3. *(100 points)* **Can CDMA Be Asynchronous?:** We have assumed that user signaling intervals are aligned so that their codewords begin and end at the same time. For multiuser systems with fixed codewords, misalignment is a problem. However, in the previous problem, we introduced a system where the sequence of bits which describe the codeword stream for a given user is essentially random – the CDMA system in the previous problem does not use fixed codewords for each user.

   Assume that this "random codeword stream" method is used and that the codewords are truly random with i.i.d. chips and that the receiver for user m IS synchronized to user m's transmissions. You may also assume that all users are "chip-synchronous" so that the chip intervals are synchronized. However, you may NOT assume that other users' transmissions are synchronized at the codeword level.

   (a) *50 points* Please carefully set up the problem definition and then derive an expression for the SNR of a given user (say user 1). You may assume that all users' transmissions are received with equal power (i.e., $a_{m_k} = 1$ for all users).

   (b) *(50 points)* Discuss the utility of your result given that there will be multiple cells in any real wireless system.

   (c) *(50 points)* Typical CDMA systems operate in a 1.25 MHz bandwidth. Please provide a rough estimate of the chip duration (and then for good measure, the bit rate assuming 128 chips/codeword). Roughly how well must chip synchronization be maintained in order for this CDMA sytem to operate properly.