Small Scale Fading in Radio Propagation

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Abstract - One of the many impairments inherently present in any wireless communication system, that must be recognised and often effectively mitigated for a system to function well, is *fading*. Fading itself has been studied and classified into a number of different types. Here we present a detailed mathematical analysis and some useful models for capturing the effect of small scale fading. Further we discuss the types of fading as per the behaviour of the wireless channel with respect to the transmit signal.

I. RAYLEIGH FADING

Small scale fading is a characteristic of radio propagation resulting from the presence reflectors and scatterers that cause multiple versions of the transmitted signal to arrive at the receiver, each distorted in amplitude, phase and angle of arrival. Consider the situation shown in Fig. 1 wherein a mobile receiver (*mobile station* or MS) is assumed to be travelling along the positive x axis with a velocity v m/s. The figure shows one of the many waves arriving at the mobile station. Let us call this the n^{th} incoming wave. Let it be incident at an angle $\theta_n(t)$, where the dependence on t stems from the fact that the receiver is not stationary.

The motion of the MS produces a Doppler shift in the received frequency as compared to the carrier frequency. This doppler offset is given by:

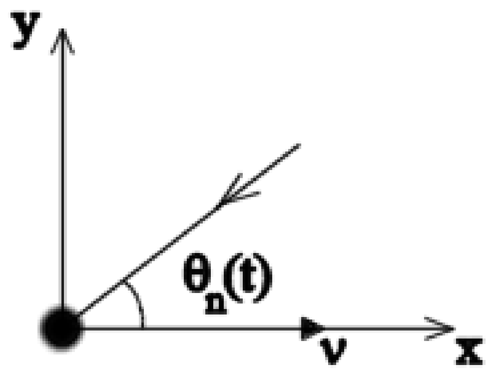

Figure 1: The figure shows a mobile station moving along the positive x-axis moving at a velocity of v m/s and the n^{th} incoming wave at an angle of $\theta_n(t)$.

$$f_{D,n}(t) = f_m \cos(\theta_n(t)) \tag{1}$$

where f_m = maximum Doppler frequency = v/λ, λ being the wavelength of the radiowave. Waves arriving from the direction of motion cause a positive doppler shift, while those coming from the opposite diection cause a negative doppler shift. We wish to derive a mathematical framework to characterize the effects of small scale fading. Consider the transmit bandpass signal:

$$s(t) = Re\{u(t).e^{j2\pi f_c t}\} \tag{2}$$

where $u(t)$ is the complex baseband equaivalent of the bandpass transmit signal. If N waves arrive at

*Taught by Dr. Narayan Mandayam, Rutgers University.

the MS, the received bandpass signal can be written as:

$$x(t) = Re\{r(t)e^{j2\pi f_c t}\} \tag{3}$$

with

$$r(t) = \sum_{n=1}^{N} \alpha_n(t).e^{-j2\pi\phi_n(t)}u(t-\tau_n(t)) \tag{4}$$

where

$$\phi_n(t) = (f_c + f_{D,n}(t))\tau_n(t) - f_{D,n}(t).t \tag{5}$$

is the phase associated with the n^{th} wave. The above expression for $r(t)$ looks like the output of a linear time-varying system. Therefore the channel can be modeled as a linear filter with a time varying impulse response given by:

$$c(\tau, t) = \sum_{n=1}^{N} \alpha_n(t)e^{-j\phi_n(t)}\delta(\tau - \tau_n(t)) \tag{6}$$

$c(\tau, t)$ is the channel response at time t to an input at time $t - \tau$. Typically the quantity $f_c + f_{D,n}(t)$ is large. This means that a small change in delay $\tau_n(t)$ causes a large change in the phase $\phi_n(t)$. The delays themselves are random. This implies that the phases of the incoming waves are random. The $\alpha_n(t)$'s are not very different from one another, i.e. the $\alpha_n(t)$'s do not change much over a small time scale. Therefore the received signal is a sum of a large number of waves with random phases. The random phases imply that sometime these waves add constructively producing a received signal with large amplitude, while at other times they add destructively, resulting in a very low amplitude. This precise effect is termed small-scale fading, and the time scale at which the resulting fluctuation of amplitude occurs is of the order of one wave-cycle of the carrier frequency. The range of amplitude variation that can result can be *upto* 60 to 70 dB. Small scale fading is therefore pramarily due to the random variations in phase $\phi_n(t)$ and also because of the doppler frequency $f_{D,n}(t)$. The effect of fading is even more important at higher data rates, as we shall see later.

Further continuing our modeling of the received signal, we can neglect the baseband modulating signal for narrowband signals (i.e. signals in which the baseband signal bandwidth is very small compared to the carrier frequency, which is true of most communication systems) and consider the unmodulated carrier alone.

$$r(t) = \sum_{n=1}^{N} \alpha_n(t)e^{-j\phi_n(t)} \tag{7}$$

$$x(t) = Re\{\sum_{n=1}^{N} \alpha_N(t)e^{-j\phi_n(t)}e^{j2\pi f_c t}\} \tag{8}$$

$$= r_I(t)\cos(2\pi f_c t) - r_Q(t)\sin(2\pi f_c t)$$

where

$$r_I(t) = \sum_{n=1}^{N} \alpha_n(t)\cos(2\pi f_c t) \tag{9}$$

$$r_Q(t) = \sum_{n=1}^{N} \alpha_n(t)\sin(2\pi f_c t) \tag{10}$$

$$r(t) = r_I(t) + r_Q(t) \tag{11}$$

$r_I(t)$ and $r_Q(t)$ are respectively the in-phase and the quadrature-phase components of the complex baseband equivalent of the received signal. Now we invoke the *Central Limit Theorem* for large N. This makes $r_I(t)$ and $r_Q(t)$ independent gaussian random processes. Further, assuming all the random processes involved are WSS, we have:

$$f_{D,n}(t) = f_{D,n} \tag{12}$$

$$\alpha_n(t) = \alpha_n \tag{13}$$

$$\tau_n(t) = \tau_n \tag{14}$$

We also assume that $x(t)$ is WSS.

$$\Phi_{xx}(\tau) = E\{x(t).x(t+\tau)\} \tag{15}$$

$$= \Phi_{r_I r_I}(\tau).\cos(2\pi f_c t) - \Phi_{r_Q r_I}(\tau).\sin(2\pi f_c t)$$

Now,

$$\Phi_{r_I r_I}(\tau) = E\{r_I(t) r_I(t+\tau)\} \tag{16}$$

$$= E\{\{\sum_{i=1}^{N} \alpha_i \cos(\phi_i t)\}.\{\sum_{j=1}^{N} \alpha_j \cos(\phi_j (t+\tau))\}\}$$

We can assume the ϕ_j's are independent because delays and doppler shifts are independent from path to path.

$$\phi_n(t) = U(-\pi, \pi) \tag{17}$$

On evaluating the expectations, we get:

$$\Phi_{r_I r_I}(\tau) = \frac{\Omega_p}{2} E\left\{\cos(2\pi f_{D,n} \tau)\right\} \tag{18}$$

where

$$\frac{\Omega_p}{2} = \frac{1}{2} \sum_{i=1}^{N} E\left\{\alpha_i^2\right\} \tag{19}$$

which is the total average received power from all multipath components. Now, in the expression above (18), we have

$$f_{D,n} = f_m \cos(\theta_n) \tag{20}$$

Therefore, we have the auto correlation function of the in-phase component $r_I(t)$:

$$\Phi_{r_I r_I}(\tau) = \frac{\Omega_p}{2} E_\theta \left\{\cos(2\pi f_m \tau \cos(\theta)\right\} \tag{21}$$

Going through a similar series of steps for the cross-correlation function between the in-phase and quadrature-phase component, we get:

$$\Phi_{r_I r_Q}(\tau) = E\left\{r_I(t) r_Q(t+\tau)\right\} \tag{22}$$

$$= \frac{\Omega_p}{2} E_\theta \left\{\cos(2\pi f_m \tau \cos(\theta))\right\}$$

If the 2-D isotropic scattering assumption is used in the above analysis (i.e. the incoming angle θ is uniformly distributed over $(-\pi, \pi)$) then the above is called the Clarke's Model. Using the uniform distribution for θ in the above, we get:

$$\Phi_{r_I r_I}(\tau) = \frac{\Omega_p}{2}.\frac{1}{2\pi} \int_{-\pi}^{+\pi} \cos(2\pi f_m \tau.\cos(\theta)) d\theta \tag{23}$$

which with a change of variable gives us:

$$= \frac{\Omega_p}{2}.\underbrace{\frac{1}{\pi} \int_{0}^{+\pi} \cos(2\pi f_m \tau.\sin(\theta)) d\theta}_{J_0(2\pi f_m \tau)} \tag{24}$$

$$= \frac{\Omega_p}{2}.J_0(2\pi f_m \tau)$$

where $J_0(.)$ is the Bessel function of the zeroth order and first kind.[1]

Similarly, using the uniform pdf for θ in the expression for cross correlation of the in-phase and quadrature phase components of $r(t)$ gives:

$$\Phi_{r_I r_Q} = 0 \tag{25}$$

We are now in a position to talk about the PSD of $r_I(t)$:

$$S_{r_I r_I}(f) = \mathcal{F}\left\{\Phi_{r_I r_I}(\tau)\right\} \tag{26}$$

$$= \begin{cases} \frac{\Omega_p}{4\pi f_m} \frac{1}{\sqrt{1-(f/f_m)^2}} & |f| < f_m \\ 0 & \text{otherwise} \end{cases}$$

[1] The Bessel functions of the first kind $J_n(x)$ are defined as the solutions to the Bessel differential equation: $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$. The bessel function $J_n(x)$ can also be defined in terms of the contour integral: $J_n(x) = \frac{1}{2\pi j} \int e^{(x/2)(t-1/t)} t^{-n-1} dt$ where the contour encloses the origin and is traversed in a counter-clockwise direction. For the special case of $n = 0$ a closed form expression due to *Frobenius* is $J_o(x) = \sum_{k=0}^{\infty} (-1)^K \frac{(\frac{1}{4}x^2)^2}{(k!)^2}$ or the integral $J_0(x) = \frac{1}{\pi} \int_0^{\pi} e^{jx\cos(\theta)} d\theta$

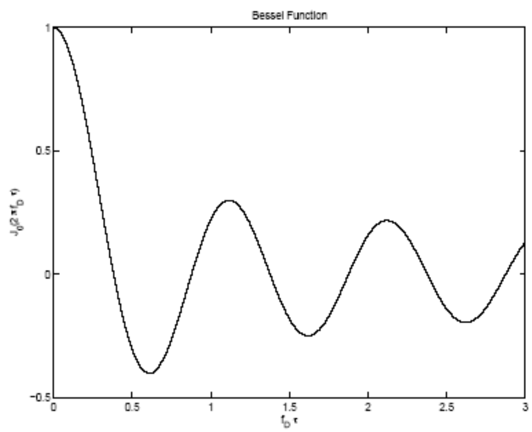

Figure 2: Bessel function of the zeroth order and the first type. This is the shape of the autocorrelation function $\Phi_{r_I r_I}(\tau)$ of the in-phase component of the complex baseband equivalent of the received signal.

Having obtained the PSD of $r_I(t)$, we can now proceed to derive the PSD $x(t)$ as follows:

$$r(t) = r_I(t) + jr_Q(t) \tag{27}$$

$$\Phi_{rr}(\tau) = E\left\{r^*(t).r(t+\tau)\right\} \tag{28}$$

$$= \Phi_{r_I r_I}(\tau) + j\Phi_{r_I r_Q}(\tau)$$

Therefore:

$$\Phi_{rr}(\tau) = \Phi_{r_I r_I}(\tau)$$

Further:

$$\Phi_{xx}(\tau) = Re\left\{\Phi_{rr}(\tau).e^{j2\pi f_c t}\right\} \tag{29}$$

$$= Re\left\{\Phi_{r_I r_I}(\tau).e^{j2\pi f_c t}\right\}$$

$$S_{xx}(f) = \mathcal{F}\left\{Re\left\{\Phi_{r_I r_I}(\tau).e^{j2\pi f_c t}\right\}\right\} \tag{30}$$

$$= \mathcal{F}\left\{\frac{\Phi_{r_I r_I}(\tau).e^{j2\pi f_c t} + \Phi^*_{r_I r_I}(\tau).e^{-j2\pi f_c t}}{2}\right\}$$

Note that $\Phi_{r_I r_I}(\tau) = \Phi^*_{r_I r_I}(-\tau)$, and so for real $r_I(t)$, $\Phi_{r_I r_I}(\tau) = \Phi_{r_I r_I}(-\tau)$. Thus we have:

$$S_{xx}(f) = \mathcal{F}\left\{\frac{\Phi_{r_I r_I}(\tau).e^{j2\pi f_c t} + \Phi_{r_I r_I}(\tau).e^{-j2\pi f_c t}}{2}\right\} \tag{31}$$

$$S_{xx}(f) = \frac{1}{2}\left\{S_{r_I r_I}(f - f_c) + S_{r_I r_I}(-f - f_c)\right\} \tag{32}$$

Now we shall make use of the knowledge that that $r(t) = r_I(t) + j.r_Q(t)$ is a complex Gaussian process for large N. Therefore the envelope $z(t) = |r(t)| = \sqrt{r_I^2(t) + r_Q^2(t)}$ has a Rayleigh distribution [2]

$$P_z(x) = \frac{x}{\sigma^2}.e^{-x^2/2\sigma^2}; x \geq 0 \tag{33}$$

where $E\{z^2\} = \Omega_p = 2\sigma^2 =$ average power. Thus we have the probability density function of the recd. signal given by:

$$P_z(x) = \frac{x}{\Omega_p/2}.e^{-x^2/\Omega_p}; x \geq 0 \tag{34}$$

The above is called Rayleigh fading and is derived from Clarke's fading model, wherein the PSD of the received signal has the U-shape shown above. Rayleigh fading is generally applicable when there is no line-of-sight component. This is a good model for cellular mobile radio. Also note that the squared envelope $|r(t)|^2$ is exponentially distributed at any time t:

$$P_{z^2}(x) = \frac{1}{\Omega_p}.e^{-x/\Omega_p}; x \geq 0 \tag{35}$$

[2] It is known that the random variable obtained by finding the square-root of the sum of the squares of two independent gaussian random variables has a Rayleigh distribution

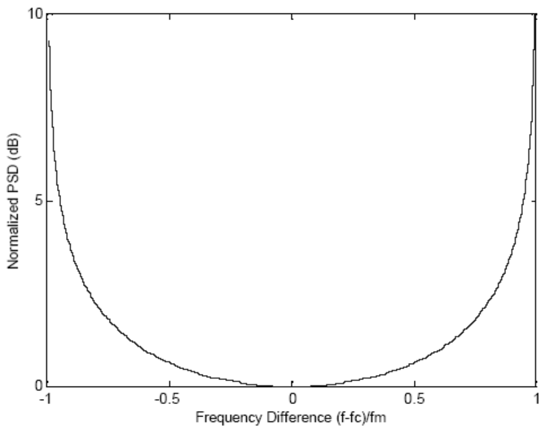

Figure 3: Power spectral density of the received signal, $S_{xx}(f)$. This is called the U-shaped PSD characteristic of Rayleigh fading modeled by the clarke's model

II. RICIAN FADING

We now consider the situation that arises when there *is* a line-of-sight component in the received signal. This is common in microcellular systems. The probability dstribution for the envelope of the received signal is then given by:

$$p_z(x) = \frac{x}{\sigma^2}.e^{\frac{-(x^2+s^2)}{2\sigma^2}}.I_0\left\{\frac{xs}{\sigma^2}\right) ; x \geq 0 \qquad (36)$$

where $s^2 = \alpha_0^2 \cos^2\theta_0 + \alpha_0^2 \sin^2\theta_0 = \alpha_0^2 =$ 'non centrality parameter'. It denotes the power in the line-of-sight component. [3] $I(.)$ is the modified bessel function of the zeroth order [4].

The quantity $K = \frac{s^2}{2\sigma^2}$ is called the Rice factor. Note that setting $K = 0$ transforms this model into the Rayleigh fading model discussed in the preceeding section and setting $K = \infty$ would transform it into a simple AWGN model with no fading. The avegrage power is given by $E[z^2] = \Omega p = s^2 + 2\sigma^2$. Also:

$$s^2 = \frac{K\omega_p}{K+1}, \quad 2\sigma^2 = \frac{\Omega_p}{K+1} \qquad (37)$$

III. NAKAGAMI FADING MODEL

The Nakagami Fading model is a purely emperical model and is not based on results derived from physical consideration of radio propagation. It uses a chi-square distrbution with m degrees of freedom. The distribution of the received signal's envelope is given by:

$$p_z(x) = \frac{2m^m x^{2m-1}}{\Gamma(m)\Omega_p^m}.e^{-mx^2/\Omega_p}; m \geq \frac{1}{2} \qquad (38)$$

where $\Omega_p = E[z^2] =$ average power, and m is a model parameter. By varying the value of this parameter, the model can capture various distributions. For $m = 1$ the model converges to the Rayleigh fading model, setting $m = 1/2$ makes it a one sided gaussian distribution, while setting $m = \infty$ transforms it into a 'no-fading' model. Finally, the Rician distribution can be approximated though the Nakagami model using the followingrelationships:

$$K = \frac{\sqrt{m^2 - m}}{m - \sqrt{m^2 - m}}; m \geq 1 \qquad (39)$$

or,

$$m = \frac{(K+1)^2}{2K+1} \qquad (40)$$

The Nakagami model is favoured because it has a closed form analytical expression.

All the small-scale fading models considered above assume that all frequencies in the transmitted signal are affected similarly by the channel, i.e. by the fading. This is called *flat-fading* or *frequency non-selective fading.*

[3] α_0 denotes the amplitude gain of the zeroth wave, (ref: previous section) which in this case is the line-of-sight component.

[4] The modified Bessel functions $I_n(x)$ are defined as the solutions to the 'modified' Bessel differential equation: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - (x^2+n^2)y = 0$ and can be expressed in terms of the Bessel functions as: $I_n(x) = (j)^{-n}J_n(jx)$

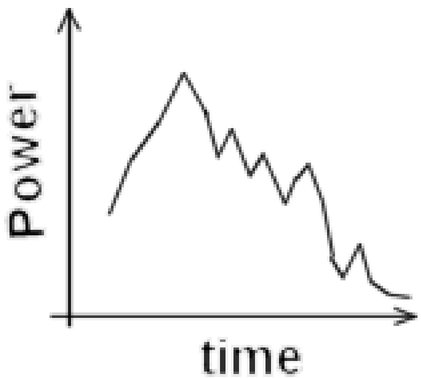

Figure 4: The power delay profile gives the the power received as a function of time when an impulse is transmitted over the wireless channel.

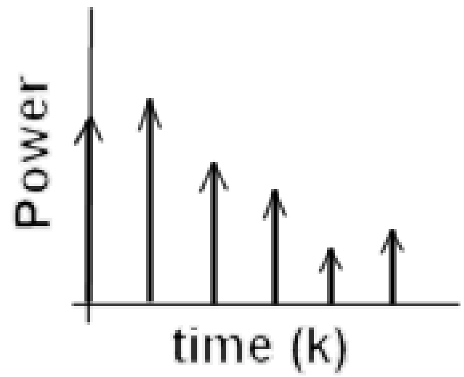

Figure 5: Since received power can only be measured on a discrete time scale, we can only have a discrete power delay profile, which indicates the power received at discrete instants of time when an impulse is transmitted on the wireless channel.

IV. FREQUENCY SELECTIVE FADING CHANNELS

Consider only WSSUS (Wide Sense Stationary Uncorrelated Scattering). Recall that the channel response is given by $c(t, \tau)$ and represents the response of the channel at time t to an input impulse at time $t - \tau$.

Definition: The *power delay profile* or *multipath intensity profile* is defined as:

$$\Phi_c(\tau) = \frac{1}{2} E[c(t,\tau)c^*(t,\tau)] \tag{41}$$

It gives the average power at the channel output as a function of time delay.

Definition: Average delay is defined as:

$$\mu_\tau = \frac{\int_0^\infty \tau \phi_c(\tau) d\tau}{\int_0^\infty \phi_c(\tau) d\tau} \tag{42}$$

Definition: RMS Delay spread is defined as:

$$\sigma_\tau = \sqrt{\frac{\int_0^\infty (\tau - \mu_T)^2 \phi_c(\tau) d\tau}{\int_0^\infty \phi_c(\tau) d\tau}} \tag{43}$$

The RMS delay spread is a way of quantifying the *multipath* nature of the channel. It is of the order of μs in outdoor situation and of the order of ns in indoor situations. Note that the absolute transmit power level does not affect the definition of σ_τ and μ_τ. Instead the above two definitions only depend on the relative amplitudes of multipath components.

As against the power delay profile shown above, in reality we can only have a discrete power delay profile. Corresponding to this discrete delay profile, we have the following definitions:

$$\tau = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)} \tag{44}$$

$$\sigma_\tau = \sqrt{\bar{\tau^2} - (\bar{\tau})^2} \tag{45}$$

where

$$\bar{\tau^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)} \tag{46}$$

V. CHARACTERIZATION OF FADING CHANNELS

Fading radio channels have been classified in two ways. [5] The first type of classification discusses whether the fading is flat (frequency non-selective) or frequency selective, while the second classification is based on the rate at which the wireless channel is changing (or in other words, the rate of change of the impulse response of channel), i.e. whether the fading is fast or slow. In connection with these characterizations of fading channels, it is useful to note the following quantities:

Coherence bandwidth: Coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be considered "flat" (i.e. frequency non-selective, or in other words a channel which passes all spectral components with equal gain and phase). It may also be defined as the range of frequencies over which any two frequency components have a strong potential for amplitude correlation. It has been shown that:

$$B_c \propto \frac{1}{\sigma_\tau} \tag{47}$$

where σ_τ is the RMS delay spread. Also, if we define the coherence bandwidth as that bandwidth over which the frequency correlationfunction is above 0.5 (i.e. the normalized cross-correlation coefficient > 0.5 for all frquencies) then $B_c \approx \frac{1}{5\sigma_\tau}$. Note that if the signal bandwidth is $> B_C$, then the different frequency components in the signal will not be faded the same way. The channel then appears to be 'frequency-selective' to the transmitted signal.

Doppler spread and Coherence time: While σ_τ and B_c describe the time dispersive nature of the channel in an area local to the receiver, they do not offer any information about the time-variations of the channel due to relative motion between the transmitter and the receiver. The doppler spread B_D, defined as a measure of spectral broadening caused by the time-rate of change of the channel (related to the doppler frequency). [6]. The coherence time is a statistical measure of the time duration over which two received signals have a strong potential for amplitude correlation. Thus if the inverse bandwidth of the basebad signal is greater than the coherence time of the channel then the channel changes during transmission of he baseband message. This will cause a distortion at the receiver. It is shown that:

$$T_c \approx \frac{1}{B_D} \tag{48}$$

If the coherence time is defined as the duration of time over which the time correlation function is > 0.5, then:

$$T_c \approx \sqrt{\frac{9}{16\pi f_m^2}} \tag{49}$$

where f_m is the maximum doppler frequency $= v/\lambda$.

Example - Consider a vehicle travelling at 60 mi. per hour and communicating with a stationary base station using a carrier frquency $f_c = 900$ Mhz. This would give a channel coherence time of $T_c \approx 6.77$ msec. Therefore if the symbol rate of transmission is greater than 150 samples per second then the fading nature of the channel doesn't really affect the transmitted signal being received by the receiver in a harmful way. For a smaller symbol rate, the symbol width is so large that the channel changes (symbol duration $> T_c$) within a single symbol.

Flat fading: If a channel has a constant response for a bandwidth $>$ the transmitted signal bandwidth, then the channel is said to be a flat fading channel. The conditions for a flat fading channel are:

$$B_s \ll B_c \tag{50}$$

$$T_s \gg T_c \tag{51}$$

[5] An excellent treatment of the characterization of fading channels is found in an article in the Sept. 1997 issue of the IEEE communications magazine: 'Rayleigh Fading Channels in Mobile Digital Communication Systems: Part I: Caharacterization', by Bernard Sklar.

[6] If the baseband signal frequency is much greater than the doppler spread B_D then the effects of doppler spread are negligible

where B_s and T_s are the signal bandwidth and the symbol duration respectively.

Frequency selective fading: A channel is said to be frequency selective if the signal bandwidth is greater than the coherence bandwidth of the channel. In such a case, different frequency components of the transmit signal undergo fading to different extents. For a frequency-selective fading situation:

$$B_s > B_c \tag{52}$$

$$T_s < T_c \tag{53}$$

The concept of *pulse-shaping* is used to control the transmit signal bandwidth. This is used in the degin of the transmit symbol such that given the required symbol rate of transmission, a pulse shape is designed so as to make the signal bandwidth fit within the coherence bandwidth of the signal. Ofcourse, this places an upper limit on the achievable symbol rate. OFDM attempts to solve this problem by breaking up the signal bandwidth into sub-carriers, each of which can be individually transmitted without the channel behaving in a frequency - selective manner. A common rule of thumb to characterize a channel as frequency selective is that if:

$$\sigma_\tau > 0.1T_s \tag{54}$$

Fast fading: In a fast fading channel, the channel impulse response changes rapidly within the symbol duration, i.e. the coherence time of the channel is smaller that the symbol period of the transmitted signal. Viewed in the frequency domain, signal distortion due to fast fading increases with increasing Doppler spread relative to the bandwidth of the transmitted signal. Therefore, a signal undergoes fast fading if:

$$T_s > T_c \tag{55}$$

$$B_s < B_D \tag{56}$$

where B_D is the Doppler spread of the channel and T_c is its coherence time.

Slow fading: In a slow fading channel, the channel impulse response changes at a rate much slower than the transmitted baseband signal $S(t)$. In the frequency domain, this implies that the Doppler spread of the channel is much less than the bandwidth of the baseband signal. There fore, a signal undergoes slow fading if:

$$T_s \ll T_c \tag{57}$$

$$B_s \gg B_D \tag{58}$$

Key Channel Parameters and Time Scales	Symbol	Typical Value
Carrier frequency	f_c	1 GHz
Communication bandwidth	W	1 MHz
Distance between Tx and Rx	d	1 km
Velocity of mobile	v	64 km/h
Doppler shift for a path	$f_m = f_c v/c$	50 Hz
Time for change in path gain	d/v	1 min
Time for change in path phase	$1/(4f_m)$	5 ms
Coherence time	$T_c = 1/(B_D)$	2.5 ms
Delay spread	σ_τ	1 μs
Coherence bandwidth	$B_c \approx 1/2\sigma_\tau$	500 kHz

Table 1: A summary of the physical parameters of the channel and the time scale of change of the key parameters in its discrete-time baseband model. (Taken from 'Fundamentals of Wireless Communication', David Tse, University of California Berkely, Promod Vishwanath, University of Illinios Urbana-champaign)

It must be noted that the wireless channel is function of what is transmitted over it. In order to determine whether fading will affect communication on a wireless channel, we must compare the symbol duration of data transmission with the coherence time and the bandwidth of the baseband signal (fast / slow fading) with the coherence bandwidth of the channel (flat / frequency selective nature).

It should also be clear that when a channel is specified as a fast or slow fading channel, it does not specify whether the channel is flat fading or frequency selective in nature. These are two independent classifications. Fast and slow fading deal with the time rate of change of the channel with reference to the transmitted signal, whicle flat and frequency-selective fading deal with weather the relationship between the signal bandwidth and the range of frequencies over which the fading behaviour of the channel is uniform.

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