Quizlet I
FOR FUN, NOT FOR CREDIT
BUT: if you don’t get a high score on this, you should worry

1. (30 points) Linear Systems:
   (a) (10 points) Once again write down the forward and reverse Fourier Transform which relates \( x(t) \) and its Fourier Transform \( X(f) \).
   (b) (20 points) Show that if \( x(t) \) has Fourier Transform \( X(f) \), then the Fourier Transform of \( x(t-t_0) \) is \( e^{-j2\pi f t_0}X(f) \).

2. (30 points) Amplitude Modulation:
   (a) (10 points) What is the Fourier Transform of \( m(t) \cos 2\pi f_c t \) given the Fourier Transform of \( m(t) \) is \( M(f) \)?
   (b) (10 points) What is the Fourier Transform of \( m(t) \cos 2\pi f_c t + jm(t) \sin 2\pi f_c t \) given the Fourier Transform of \( m(t) \) is \( M(f) \)?
   (c) (10 points) The previous part is an (unrealizable) form of what sort of modulation?

3. (30 points) Quantization:
   (a) (10 points) What is the purpose of a quantizer? State your answer in words (no more than a short paragraph). NOTE: this is not an optimality question, just a simple question about what a quantizer is used for.
   (b) (10 points) The Loyd-Max conditions for optimal quantization are \( q_k = E[X|X \in A_k] \) where \( A_k \) is the event that random variable \( X \in (x_{k-1}, x_k) \) and \( x_k = \frac{1}{2}(q_k + q_{k+1}) \). Suppose \( f_X(x) = [u(x+1) - u(x-1)]/2 \). Is a 1 bit quantizer with \( q0 = -0.5 \), \( q1 = 0.5 \) and \( x0 = 0 \) optimal? Why/Why not?
   (c) (10 points) Sketch the output to this quantizer on the interval \( t \in (0, 6) \) when the input is the sawtooth waveform
   \[
   m(t) = u_{-2}(t) + 2 \sum_{k=0}^{\infty} (-1)^k u_{-2}(t - 2k + 1)
   \]
   where \( u_{-2}(t) \) is the unit ramp (the integral of the unit step). Then provide an analytic expression for \( Q(m(t)) \) in terms of the unit step function \( u(t) \) (also known as \( u_{-1}(t) \) in some circles).