RUTGERS

School of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring 2011**
Quiz II

There are **THREE** problems. Each problem subpart is stated on a different sheet. Show all work on the stapled sheets provided (front and back). DO NOT DETACH THE SHEETS. You are allowed two sides of an $8.5 \times 11\text{in}^2$ paper handwritten note sheet.

1. *(50 points)* **Quantization:**

   (a) *(25 points)* The amplitude of a random signal $m(t)$ is uniform on $(0, A)$. Please provide an OPTIMAL (minimum mean square error) 2-bit quantizer for this signal. Your answer should be given as a carefully labeled sketch along with mathematical justification that your quantizer is optimum.

   (b) *(25 points)* Using a series of shell corporations to obscure her identity, Cora the communication engineer has won the bid to build a quantizer for MTS, Inc, Marty the Squirrel's startup company. The quantizer is to be used in the MTS *Squirrel Defense System* and needless to say given their history, Cora wants to screw things up maximally. Assume that the input signal to the quantizer is uniform on $(-1, 1)$ and that 2 bits of quantization are required.

   What quantization function will result in MAXIMUM mean square error, assuming that the quantization level values q_I are restricted to lie in $(-1, 1)$? Please sketch your result and justify it mathematically.

2. *(50 points)* **Cora and the Squirliminator:** Cora has decided to annoy Marty the squirrel using a laser beam. Because Cora lost her security clearance in a previous failed attempt to spike Marty's nuts with plutonium, she cannot obtain weapons grade lasers and has to resort to using laser pointers whose beams are so weak that in order to even annoy Marty, they must be kept on target for minutes at a time.

   Cora observes Marty and notices that he uses a particular telephone line to reach his foraging tree. She decides to set up her laser system across the street and mounts it on a servo motor she bought using her Amazon.com account. Servo motors move in step increments and the servo controller she uses can be pulsed at a maximum rate of f_s. She tests the system and each '1' sent to the servo results in beam movement Δ along the wire and each '0' results in $-\Delta$ movement where Δ is much smaller than Marty's length. These are the ONLY two signals that can be delivered to the servo.

   Cora uses three beams, from left to right, Red, Green and Blue. The central power beam is green. The left and right beams, red and blue respectively, are sensing beams. If there is no reflection from the red beam, then Cora's system knows that Marty is right of power beam center. If there is no reflection from the blue beam, then Marty is left of power beam center.

The three beam "fan" is designed so that the red and blue beams can never be on Marty at the same time (their spread is *just* longer than Marty). If there is no reflection at all, the her system goes into target acquisition mode (sweeping right and left) until Marty is acquired.

(a) *(10 points)* Assuming Marty has been acquired, please draw a labeled block diagram of Cora's system. Is this system similar to a delta modulator? Why/why not?

(b) *(10 points)* Assume Marty's position is $x(t)$ along the wire and that he can move at maximum velocity v. What values of f_s and Δ will guarantee that once Marty is acquired by the system, the green power beam stay on Marty (not necessarily dead center, and assuming Marty stays on the wire, of course)?

(c) *(10 points)* Assume that Marty, groggy from hibernation, freezes when he is first caught in the power beam. Please sketch the signal sent to the servo by Cora's system. You must justify your answer.

(d) *(10 points)* Suppose either f_s OR Δ (but not both) can be adjusted on-the-fly in Cora's system and that Marty's maximum velocity is unknown. Describe in words (or even better, psuedo-code) a tracking algorithm that will keep the beam as closely as possible on Marty. You must be explicit, using the available feedback and control to drive your algorithm.

(e) *(10 points)* Unbeknownst to Cora, Marty is blazingly fast and after his initial shock, easily outruns the beam. For revenge, he taps into Cora's servo signal and attaches a bomb which will detonate when it receives the control sequence 1111000011110000. Assuming Marty knows f_s and Δ and that Δ is much smaller than Marty's length, please provide a labeled sketch of the $x(t)$ he should execute to detonate the bomb. Is this $x(t)$ unique (the only one that Marty can run and still detonate the bomb)?

3. *(50 points)* **Chimeric Modulation:**

Let

$$p(t) = \cos(2\pi f_c t + \beta \sin 4\pi B t)$$

where $B = f_c/1000$ and $f_c > 0$.

(a) *(10 points)* Is $p(t)$ periodic? If so, provide its period. If not, show why not.

(b) *(10 points)* Suppose program material $m(t)$ has bandwidth $\pm B$ centered around $f = 0$ and we form $r(t) = m(t)p(t)$. What type of modulation best describes $r(t)$?

(c) *(10 points)* Please sketch an approximation to the spectrum of $r(t)$ for $\beta = 5$. (You may use any generic shape your choosing for the baseband spectrum $M(f)$.)

(d) *(20 points)* What is the minimum value of β which will allow recovery of $m(t)$ when $r(t)$ is passed through a perfect low pass filter with bandwidth $\pm B$? State all assumptions.

HINT: Your result can be proportional to $m(t)$ as opposed to exactly equal to $m(t)$.