THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring 2008**

Quiz II

There are 4 questions. You have the class period to answer them. Show all work. Answers given without work will receive no credit. GOOD LUCK!

1. *(30 points)* **Sampling from Hell:** There's a minor problem with the sampling theorem as usually presented (and we've been no exception). This problem will help you correct it.

   (a) *(10 points)* A signal $m(t)$ has bandwidth $20kHz$. What minimum sampling rate f_s will allow $m(t)$ to be recovered from the samples $\{m(\frac{k}{f_s})\}$ where k is an integer?

   (b) *(10 points)* For A some constant, we sample $s(t) = A\sin(2\pi f_0 t)$ at times $t \in \{\frac{k}{2f_0}\}$. What are the sample values? Can $s(t)$ be reconstructed from these samples?

   (c) *(10 points)* Sketch the spectrum of

   $$q(t) = s(t)\sum_k \delta(t - \frac{k}{f_s})$$

   for $s(t) = \sin(2\pi f_0 t)$ where f_s is the sampling rate. Identify the problem with the sampling theorem implied by the previous part and then rewrite the sampling theorem correctly.

2. *(30 points)* **Quantization from Hell:**

   The PDF of signal levels attained by a signal $x(t)$ is

   $$f_X(x) = \frac{1}{4}\delta(x+3) + \frac{1}{4}\delta(x+1) + \frac{1}{4}\delta(x-1) + \frac{1}{4}\delta(x-3)$$

   (a) *(10 points)* Please derive an optimal 2-bit quantizer for the signal $x(t)$. What is the expected error of your quantization function?

   (b) *(20 points)* Please derive an optimum 1-bit quantizer for $x(t)$. What is the expected error of your quantization function?

3. *(40 points)* **Phase Locked Loop from Hell:**

   An unlabeled block diagram of a phase-locked loop is shown in FIGURE 1. $\dot{\hat{\theta}}(t)$ is the derivative of $\hat{\theta}(t)$, an estimate of $\theta(t)$.

   (a) *(10 points)* What is the block labeled A?

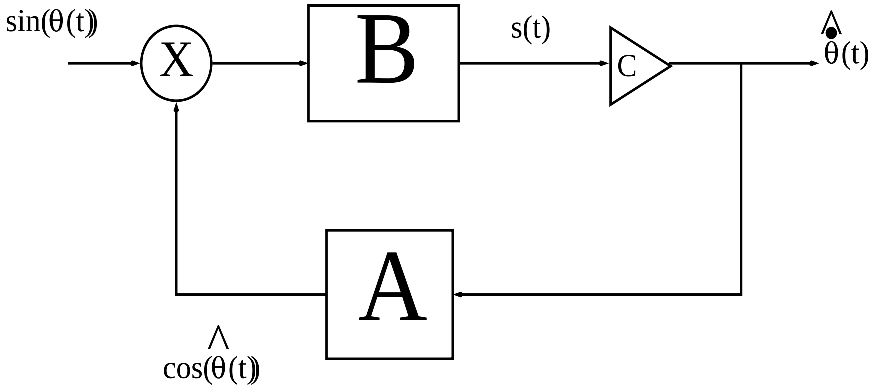

Figure 1: Phase locked loop diagram for problem 3

(b) *(10 points)* What is the block labeled B?

(c) *(10 points)* What is the block labeled C?

(d) *(10 points)* Explain the operation of the phase locked loop in detail.

4. *(50 points)* **Cora and Marty, the Demon Squirrel from Hell:** Cora the Communications Engineer needs to guard against Marty, the demon squirrel from hell. Marty pops up at position X_n and sets fire to whatever's there. Cora has to put the fire out, but her hose is heavy and if it's pointed in the wrong direction, she won't douse the flames in time. So, Cora needs to predict where Marty will be. It should come as no surprise that Cora will use a linear predictor structure for this purpose.

Unbeknownst to Cora, Marty is a creature of habit whose firing positions follow

$$X_n = \frac{1}{2}X_{n-1} + G_n$$

where the $\{G_n\}$ are i.i.d. zero mean Gaussian random variable with unit variance. Cora will use a simple one-step estimate $\hat{X}_n = wX_{n-1}$ you need to provide her with the w which minimizes $\varepsilon^2 = E\left[(X_n - \hat{X}_n)^2\right]$.

(a) *(10 points)* Please derive an expression for Marty's position based on all past inputs G_k and his initial position X_0. Show that in the limit of large n, Marty's initial position is irrelevant.

(b) *(10 points)* X_n is a sequence of random variables owing to the random inputs G_n. For very large n, what is $E[X_n]$, what is $E[X_n^2]$, what is $f_{X_n}(x)$?

(c) *(10 points)* Find an expression for $E[X_n X_{n-\ell}]$ the correlation function of the random sequence X_n. Show that $E[X_n X_{n-\ell}] = R_X(\ell)$ does not depend on n for n large.

(d) *(10 points)* Cora can form a good estimate of the correlation function $R_X(\ell)$. What w minimizes ε^2? Is it the same as or different from $\frac{1}{2}$, the coefficient from Marty's equation of motion? What is the resultant error, ε^2.

(e) *(10 points)* What are the weights w_1 and w_2 if Cora uses a two-step predictor $\hat{X}_n = w_1X_{n-1} + w_2X_{n-2}$? What is the error, ε^2?