THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring 2005**

Quiz I

There are four questions. You have the class period to answer them. Show all work. Answers given without work will receive no credit. GOOD LUCK!

1. *(50 points)* **Linear Systems Quickie Melange:**

   (a) *(10 points)* Provide Fourier SERIES expansions for $\cos(2\pi t)$, $\sin(2\pi t)$ and $\cos(2\pi t) + \cos(6\pi t)$.

   (b) *(10 points)* Provide Fourier TRANSFORMS for $\cos(2\pi t)$, $\sin(2\pi t)$ and $\cos(2\pi t) + \cos(6\pi t)$.

   (c) *(10 points)* Provide the Fourier TRANSFORM of $\cos^2(2\pi t)$.

   (d) *(10 points)* Find the energy in $x(t) = \frac{\sin 2\pi t}{\pi t}$.

   (e) *(10 points)* $x(t) = \frac{\sin 2\pi t}{\pi t}$ is applied to a filter with impulse response $h(t) = \frac{\sin 2\pi t}{\pi t}$. What is the output $y(t) = (x * h)(t)$?

2. *(50 points)* **Amplitude Modulation:** You are given two signals, $m_1(t)$ and $m_2(t)$ with spectra $M_1(f)$ and $M_2(f)$ as shown in FIGURE 1. For all parts of this problem you can assume you

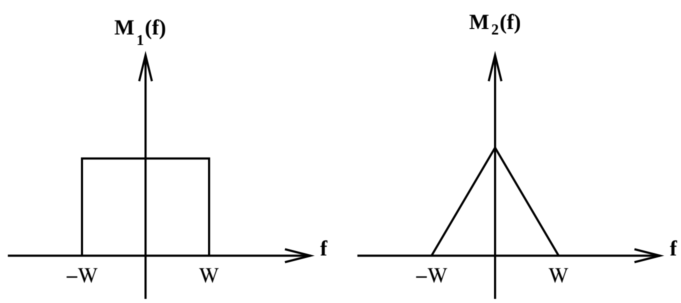

Figure 1: Spectra for problem 2.

   have a modulation/demodulation toolkit which contains as many multipliers as you'd like and as many linear time invariant filters as you'd like (all ideal). You must also assume that the signals $m_1(t)$ and $m_2(t)$ must be sent simultaneously.

   (a) *(15 points)* You are given oscillators which output $\cos(2\pi f_c t)$ and $\cos(4\pi f_c t)$ where $f_c \gg W$. You may assume you have access to the same oscillators at the receiver, that the phases of the oscillators at the receiver and transmitter match, and that there is no propagation-induced phase shift of the transmitted carrier at the receiver.

Please draw carefully labeled block diagrams of an AM transmitter and an associated AM receiver which provide (possibly scaled) copies of $m_1(t)$ and $m_2(t)$ at the receiver output.

(b) *(15 points)* Now assume you are given oscillators which output $\cos(2\pi f_c t)$ and $\cos(4\pi f_c t)$ where $f_c \gg W$ but that no copies of the oscillators exist at the receiver. For this part you may assume you have access to ideal diodes.

Please draw carefully labeled block diagrams of an AM transmitter and an associated AM receiver which provides (possibly scaled) copies of $m_1(t)$ and $m_2(t)$ at the receiver output.

(c) *(20 points)* Now assume you are given oscillators which output $\cos(2\pi f_c t)$ and $\cos(2\pi(f_c + W)t)$ where $f_c \gg W$, but otherwise assume the conditions of part 2a.

Please draw carefully labeled block diagrams of an AM transmitter and an associated AM receiver which provides (possibly scaled) copies of $m_1(t)$ and $m_2(t)$ at the receiver output.

3. *(50 points)* **Cora's Boxed PLL:** Cora the Communications Engineer has been charged with setting up a phase locked loop for her employer. She has taken the PCS course a number of times and knows all about phase locked loops using sinusoids. However, her employer, Boxomatic, is in the middle of its "Box the world!" campaign and tells Cora to use only "boxy" signals. Thus, the input to the phase locked loop (see FIGURE 7) is $c(2\pi f_c t) = \mathrm{sgn}(\cos 2\pi f_c t)$ and the voltage-controlled oscillator output is $s(\phi(t))$ where $s(t) = \mathrm{sgn}(\sin t)$.

   Please show that in the limit of large Γ as given in FIGURE 7 that $\phi(t) \approx 2\pi f_c t$. State all assumptions and approximations.

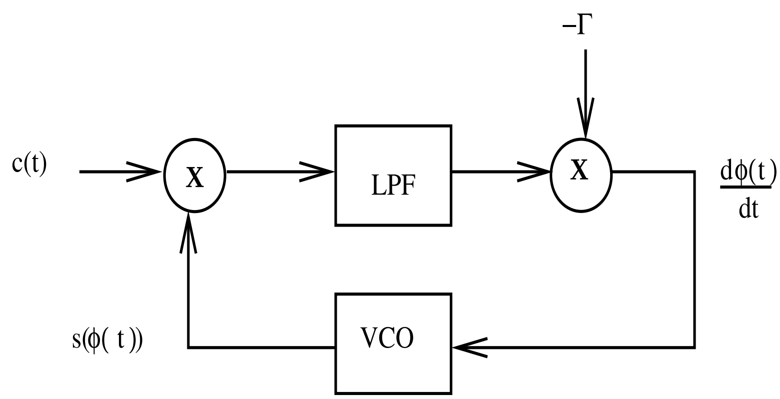

Figure 2: Phase locked loop diagram for problem 3.

4. *(50 points)* **Frequency/Phase Modulation:**

   (a) *(25 points)* Let $r(t) = A\cos(2\pi f_c t + \beta m(t))$ where A is a constant. For $|\beta| \ll 1$ provide an approximate analytic expression for the spectrum of $r(t)$.

   (b) *(25 points)* Let $r(t) = A\cos(2\pi f_c t + \beta f(t))$ where A is a constant. PROVE that the average power in the signal $r(t)$ is $A^2/2$ for periodic $f(t)$ independent of the value of β. You may assume that the period T of $f(t)$ obeys $f_c \gg 1/T$.