THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring 2004**

Quiz I

There are three questions. You have the class period to answer them. Show all work. Answers given without work will receive no credit. GOOD LUCK!

1. *(50 points)* **Amplitude Modulation:**

   (a) *(20 points)* Carefully sketch and envelope detector for and AM receiver and describe the purpose of each component. You may assume ideal components. Analytically describe the output of the envelope detector when used with an AM signal $m(t)\cos 2\pi f_c t$ where $m(t)$ is both positive and negative as a function of t?

   SOLUTION: *The diode rectifies the incoming signal and the resistor allows the neg-*

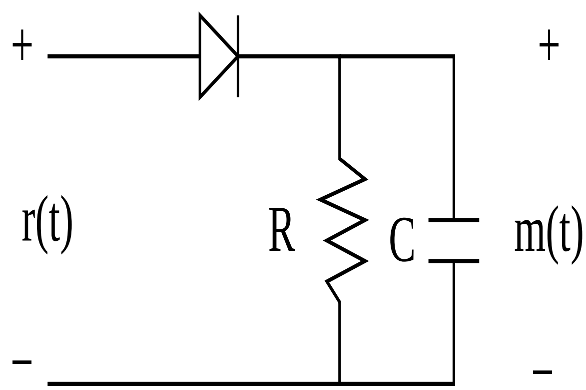

Figure 1: Envelope detector

   ative end of the diode to rise above ground. The resistor and capacitor in parallel form a low pass filter to remove the high frequency rectified carrier signal. And if $m(t)\cos 2\pi f_c t$ is passed through the envelope detector then in general $|m(t)|$ comes out because the envelope is rectified as well.

   (b) *(20 points)* In the previous part, what happens if you pass $r(t) = m(t)\sum_{k=1}^{\infty} J_k(100)\cos 2\pi k f_c t$ (where $J_k(\beta)$ is a bessel function) through the envelope detector?

   SOLUTION: *You get $|m(t)|$ as before because after rectification, all the high freqency components are filtered out and you're left with the envelope*

   (c) *(10 points)* Determine analytically and exactly the power in a signal $r(t) = m(t)\cos 2\pi f_c t$ where $m(t)$ is a signal bandlimited to $\pm W$ and $f_c \gg W$. You may assume the power in $m(t)$ is P. This is one of the few cases where scale factors matter, so you are fore-warned.

SOLUTION: *You could integrate $r(t) = m(t)\cos 2\pi f_c t$ but you're not given anything explicit for $m(t)$. However, we know that the spectrum of $r(t)$ is $R(f) = \frac{1}{2}M(f+f_c) + \frac{1}{2}M(f-f_c)$ so the power spectrum is $|R(f)|^2 = \frac{1}{4}|M(f+f_c)|^2 + \frac{1}{4}|M(f-f_c)|^2$. Then since the power in $m(t)$ is known to be P, using parsevals theorem we have $\int_{-\infty}^{\infty}|M(f \pm f_c)|^2 df = P$. So, the power in $r(t)$ is $P/2$.*

2. *(50 points)* **Frequency Modulation:**

   (a) *(20 points)* Carefully sketch $r(t) = \cos(20000\pi t + \pi m(t))$ where $m(t)$ is a square wave which only takes on values ± 1, has 50% duty cycle and period $T = 200\mu s$. What kind of modulation is this and can $m(t)$ be recovered from $r(t)$?

   HINT: Be VERY CAREFUL with your sketch and don't make assumptions about what you think the answer SHOULD be.

   SOLUTION: *This is a case of phase modulation. However, if you did your sketch carefully, you found that $r(t)$ is just a sinusoid with no phase changes at all. Therefore it carries absolutely no discernable information and $m(t)$ cannot be recovered from $r(t)$.*

   (b) *(30 points)* Suppose you're given two signals $r_1(t) = \cos(2\pi f_c t + \beta \sin 200\pi f_m t)$ and $r_2(t) = \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$ which comprise a transmitted signal $r(t) = r_1(t) + r_2(t)$. Assume that $f_c \gg 100 f_m$ and $\beta = 10$. Is it possible to recover the program content of $r_1(t)$ and $r(2(t)$ separately? If so, how? If not, why not? Does your answer change if $\beta = 200$?

   HINT: It might help to make a reasonably careful sketch of $R(f)$.

   SOLUTION: *This problem mixed two concepts. The first is wideband FM and Carson's rule. The second is the ability to see spectrally what's going on. What we see is that $R_1(f)$ and $R_2(f)$ do not overlap in frequency domain for $\beta = 10$ except at f_c. That is, the sidebands for $r_1(t)$ start at $f_c \pm 100 f_m$ whereas those for $r_2(t)$ start at $f_c \pm f_m$. Furthermore, with $\beta = 10$, the sidebands of $r_2(t)$ extend only up to about $f_c \pm 11 f_m$. So, you can recover the signals independently by applying a filter which cuts the carrier component at f_c in half and filters out either everything in the range $f_c \pm 12 f_m$ to get the information component of $r_2(t)$, or passes only signal components in that range to get the information component of $r_1(t)$.*

   The answer does change for $\beta = 200$ since now the spectra DO overlap. So strictly speaking you cannot recover the signals independently. HOWEVER, since a feature of wideband FM (and $\beta = 200$ is MAJOR LEAGUE wideband FM) is interference suppression and the spectra overlap at only a few points, f_c, $f_c \pm 100 f_m$ and $f_c \pm 200 f_m$, an argument can be made that both information signals could be recovered almost independently. For $r_1(t)$ we filter out all components of $R(f)$ on $f_c \pm 200 f_m$ except for the carrier component at f_c which we cut in half rather than remove. For $r_2(t)$ we PASS only components in that range, except for again halving the carrier component.

3. *(50 points)* **Cora's PLL:**

   As you have probably learned by now, Cora is sometimes rather contrary, though I prefer to think she's a free thinking. So, when given the problem of designing a PM demodulator for a signal $r(t) = \cos(2\pi f_c t + m(t))$, she decided to modify the usual PLL design and produce her own as shown in FIGURE 3. As you can see, Cora has replaced the usual VCO with one

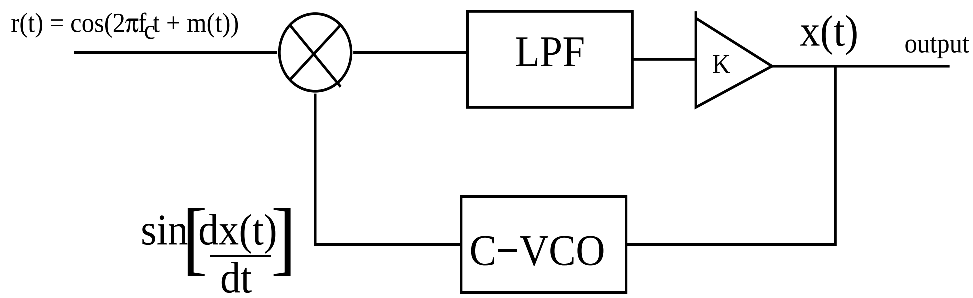

Figure 2: Cora's PLL

of her own design which in response to an input $x(t)$ produces an output $\sin\dot{x}(t)$ where $\dot{x}(t)$ is the first derivative of $x(t)$.

Please analyze Cora's PLL and determine whether it can be used as a PM demodulator. State all assumptions carefully and justify all steps of your analysis. You can choose the size and sign of K and the LPF characteristics as well.

SOLUTION: *We make the assumption that* $\dot{x} = 2\pi f_c t + m(t) + e(t)$ *with* $e(t)$ *small so that the output of the low pass filter is* $\frac{e(t)}{2}$ *which means* $x(t) = Ke(t)/2$. *This leads to* $\frac{K}{2}(\dot{x}(t) - 2\pi f_c t - m(t)) = x(t)$ *which we rewrite as*

$$\dot{x}(t) - \frac{2}{K}x(t) = (2\pi f_c t + m(t))$$

So, we can choose $K < 0$ *and the homogenous response (*$e^{\frac{2}{K}t}$*) will die down. If we choose* $|K|$ *really large then we have approximately*

$$\dot{x}(t) \approx (2\pi f_c t + m(t))$$

but we immediately see two problems. First, the homogeneous response will decay VERY slowly (even though it's exponential), so we can't be assured that the homogeneous response will die down right away. Second, the only thing available to us in the block diagram is actually $x(t)$ *which is the INTEGRAL of* $\dot{x}(t)$. *So, Cora's original notion doing direct PM demodulation is flawed. In fact, Cora will be open to LAWSUITS when her receiver electrocutes some poor PM listener – that* $2\pi f_c t$ *term gets large FAST so its integral (*$x(t)$*) gets even larger faster! But that's a small price to pay for free thinking!*