Problem Set 9 # 4

$$X(t) \longrightarrow \boxed{h_1(t)} \xrightarrow{V(t)} \boxed{h_2(t)} \xrightarrow{Y(t)}$$

* $X(t)$ is a stationary process

$$h(t) = \int_{-\infty}^{\infty} h_1(t) h_2(t-\lambda)\, d\lambda$$

$h_1(t)$ and $h_2(t)$ can be combined into 1 filter with transfer fxn $h(t)$

$$X(t) \longrightarrow \boxed{h(t)} \xrightarrow{Y(t)}$$

Lets observe $Y(t)$ at two different times t and u

This is defined by the autocorrelation function $E[Y(t)Y(u)]$

$$R_Y(t,u) = E\left[\int_{-\infty}^{\infty} h(\tau_1) X(t-\tau_1)\, d\tau_1 \int_{-\infty}^{\infty} h(\tau_2) X(u-\tau_2)\, d\tau_2 \right]$$

* As long as $E[X^2(t)]$ is finite for all t ; stable

$$R_Y(t,u) = \int_{-\infty}^{\infty} h(\tau_1)\, d\tau_1 \int_{-\infty}^{\infty} h(\tau_2)\, d\tau_2\, E[X(t-\tau_1) X(u-\tau_2)]$$

$$E[X(t-\tau_1)X(u-\tau_2)] = R_X(t-\tau_1, u-\tau_2)$$

$$R_Y(t,u) = \int_{-\infty}^{\infty} h(\tau_1)\, d\tau_1 \int_{-\infty}^{\infty} h(\tau_2)\, d\tau_2\, R_X(t-\tau_1, u-\tau_2)$$

* Since $X(t)$ is stationary ; the $R_X(t-\tau_1, u-\tau_2)$ is only a function of the difference between the times $t-\tau_1$ and $u-\tau_2$

Letting $\tau = t-u$ $\therefore$ $t-\tau_1-u+\tau_2 = \tau-\tau_1+\tau_2$

My soln $\Rightarrow$ $$R_Y(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1) h(\tau_2) R_X(\tau-\tau_1+\tau_2)\, d\tau_1\, d\tau_2$$

Soln Set $\Rightarrow$ $$R_Y(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1) h(\tau_2) R_X(\tau-\tau_1-\tau_2)\, d\tau_1\, d\tau_2$$