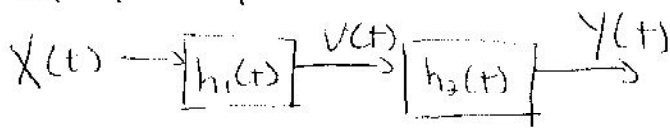


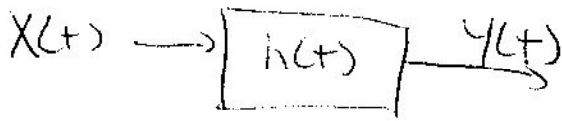
Problem Set 9 #4



\*  $X(t)$  is a stationary process

$h_1(t)$  and  $h_2(t)$  can be combined into 1 filter with transfer function  $h(t)$

$$h(t) = \int_{-\infty}^{\infty} h_1(t) h_2(t - \tau) d\tau$$



Let's observe  $Y(t)$  at two different times  $t$  and  $u$

This is defined by the autocorrelation

$$R_Y(t, u) = E \left[ \int_{-\infty}^{\infty} h(\tau_1) X(t - \tau_1) d\tau_1 \int_{-\infty}^{\infty} h(\tau_2) X(u - \tau_2) d\tau_2 \right]$$

\* As long as  $E[X^2(t)]$  is finite for all  $t$  ; stable

$$R_Y(t, u) = \int_{-\infty}^{\infty} h(\tau_1) d\tau_1 \int_{-\infty}^{\infty} h(\tau_2) d\tau_2 E[X(t - \tau_1) X(u - \tau_2)]$$

$$E[X(t - \tau_1) X(u - \tau_2)] = R_X(t - \tau_1, u - \tau_2)$$

$$R_Y(t, u) = \int_{-\infty}^{\infty} h(\tau_1) d\tau_1 \int_{-\infty}^{\infty} h(\tau_2) d\tau_2 R_X(t - \tau_1, u - \tau_2)$$

\* Since  $X(t)$  is stationary ; the  $R_X(t - \tau_1, u - \tau_2)$  is only a function of the difference between the times  $t - \tau_1$  and  $u - \tau_2$

Letting  $\tau = t - u$  ;  $t - \tau_1 - u + \tau_2 = \tau - \tau_1 + \tau_2$

My soln  $\Rightarrow R_Y(\tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\tau_1) h(\tau_2) R_X(\tau - \tau_1 + \tau_2) d\tau_1 d\tau_2$

Soln set  $\Rightarrow R_Y(\tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\tau_1) h(\tau_2) R_X(\tau - \tau_1 - \tau_2) d\tau_1 d\tau_2$