THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring**

Problem Set 9

Haykin: 1.1–1.10

1. Consider a random process $X(t)$ defined by

$$X(t) = sin(2\pi f_c t)$$

in which the frequency f_c is a random variable uniformly distributed over the range $[0, W]$. Show that $X(t)$ is nonstationary. Hint: Examine specific sample functions of the random process $X(t)$ for the frequency $f = W/2,\ W/4$ and W say.

2. Let X and Y be statistically independent Gaussian-distributed random variables each with zero mean and unit variance. Define the Gaussian process

$$Z(t) = X\ cos(2\pi t) + Y\ sin(2\pi t)$$

   (a) Determine the joint probability density function of the random variables $Z(t_1)$ and $Z(t_2)$ obtained by observing $Z(t)$ at times t_1 and t_2 respectively.

   (b) Is the process $Z(t)$ stationary? Why?

3. The square wave $x(t)$ of FIGURE 1 of constant amplitude A, period T_0, and delay t_d, represents the sample function of a random process $X(t)$. The delay is random, described by the probability density function

$$f_{T_D}(t_d) = \begin{cases} \dfrac{1}{T_0} & \frac{-T_0}{2} \leq t_d \leq \frac{T_0}{2} \\ 0 & \text{otherwise} \end{cases}$$

   (a) Determine the probability density function of the random variable $X(t_k)$ obtained by observing the random process $X(t)$ at time t_k.

   (b) Determine the mean and autocorrelation function of $X(t)$ using ensemble-averaging

   (c) Determine the mean and autocorrelation function of $X(t)$ using time-averaging.

   (d) Establish whether or not $X(t)$ is stationary. In what sense is it ergodic?

4. Consider two linear filters connected in cascade as in FIGURE 2. Let $X(t)$ be a stationary process with autocorrelation function $R_X(\tau)$. The random process appearing at the first filter output is $V(t)$ and second filter output is $Y(t)$.

   (a) Find the autocorrelation function of $Y(t)$

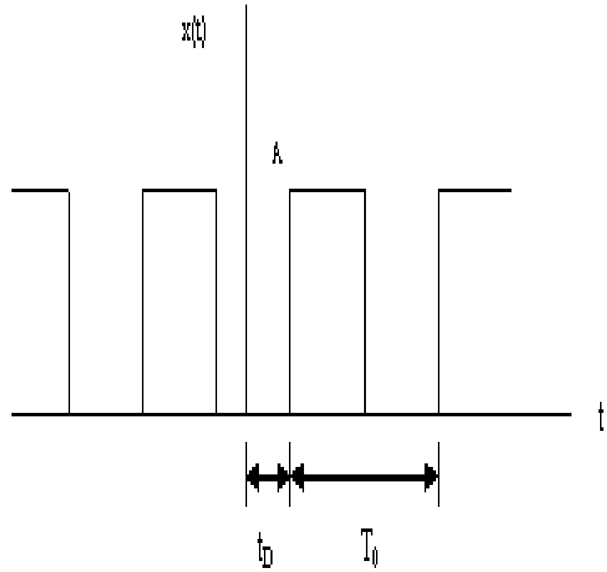

Figure 1: Square wave for $x(t)$

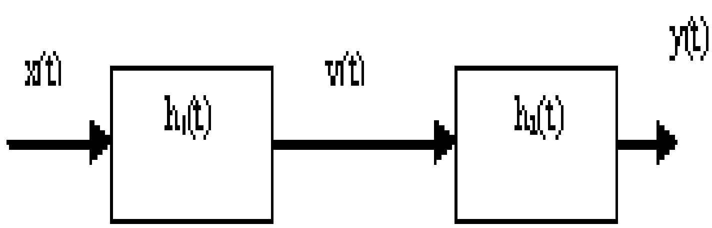

Figure 2: Cascade of linear filters

(b) Find the cross-correlation function $R_{VY}(\tau)$ of $V(t)$ and $Y(t)$.

5. A random telegraph signal $X(t)$, characterized by the autocorrelation function

$$R_X(\tau) = \exp(-2v|\tau|)$$

where v is a constant, is applied to a low-pass RC filter of FIGURE 3. Determine the power spectral density and autocorrelation function of the random process at the filter output.

6. A stationary Gaussian process $X(t)$ has zero mean and power spectral density $S_X(f)$. Determine the probability density function of a random variable obtained by observing the process $X(t)$ at some time t_k.

7. A stationary Gaussian process $X(t)$ with mean μ_x and variance σ_X^2 is passed through two linear filters with impulse responses $h_1(t)$ and $h_2(t)$, yielding processes $Y(t)$ and $Z(t)$, as shown in FIGURE 4

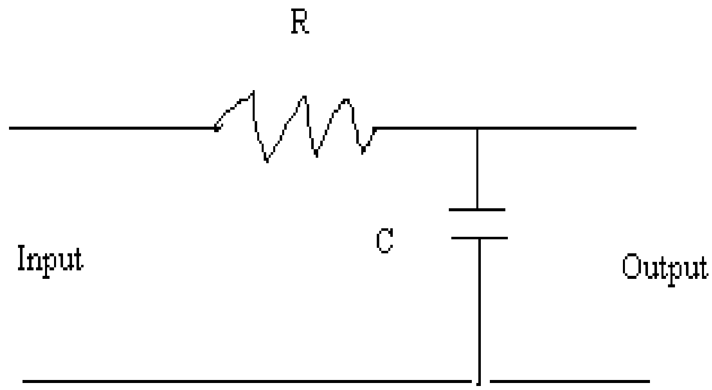

Figure 3: RC Filter

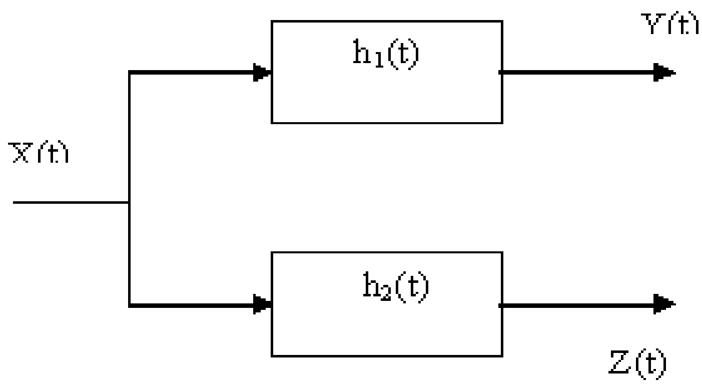

Figure 4: Parallel systems.

(a) Determine the joint probability density function of the random variables $Y(t_1)$ and $Z(t_2)$.

(b) What conditions are necessary and sufficient to ensure that $Y(t_1)$ and $Z(t_2)$ are statistically independent?

8. A stationary Gaussian process $X(t)$ with zero mean and power spectral density $S_X(f)$ is applied to a linear filter whose impulse response $h(t)$ is shown in FIGURE 5. A sample Y is taken of the random process at the filter output at time T.

   (a) Determine the mean and variance of Y

   (b) What is the probability density function of Y?

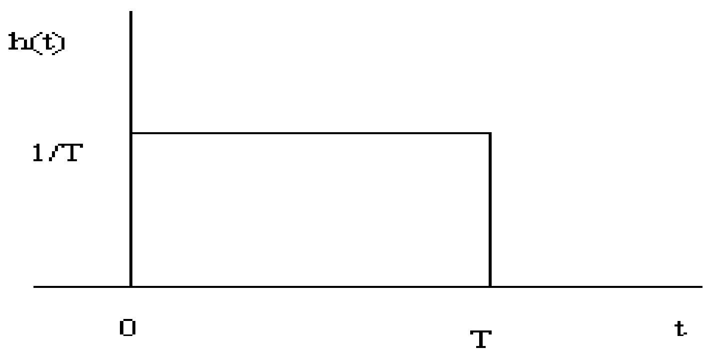

Figure 5: $h(t)$ for problem 8