THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring**

Problem Set 6

Reading:

Review Probability (appendix and 322:321 notes/text)
Web notes on convexity

1. **Counting and Sample Space:** Consider a binary code with 5 bits (0 or 1) in each code word. An example of a code word is 01010. In each code word, a bit is zero with probability 0.8, independent of any other bit.

   (a) What is the probability of the code word 00111?

   (b) What is the probability that a code word contains exactly three ones?

2. **Simple Probability, Simple Application:** A source wishes to transmit radio packets to a receiver over a radio link. The receiver uses error detection to identify packets that have been corrupted by radio noise. When a packet is sent error free the receiver sends an acknowledgement(ACK) back to the source. When the receiver gets a packet with errors, it sends back a negative-acknowledgement(NACK). Each time the source receives a NACK, the packet is re-transmitted. We assume that each packet transmission is independently corrupted by errors with probability q.

   (a) Find the PMF of X, the number of times that a packet is transmitted by the source.

   (b) Suppose each packet takes 1 millisecond to transmit and that the source waits an additional millisecond to receive the acknowledgement message (ACK or NACK) before transmitting. Let T equal the time required until the packet is successfully received. What is the relationship between T and X? What is the PMF of T?

3. **Joint PMFs:** Calls arriving ar a telephone switch are either voice calls (v) or data calls (d). Each call is a voice call with probability p, independent of any other call. Observe calls at a telephone switch until you observe two voice calls. Let M equal the number of calls up to and including the first voice call. Let N equal the number of calls observed up to and including the second voice call. Find the conditional PMF's $P_{M|N}(m|n)$ and $P_{N|M}(n|m)$. Interpret your results.

4. **Gaussians:** W is a Gaussian random variable with expected value $\mu = 0$ and variance $\sigma^2 = 16$. Given the event $C = \{W > 0\}$,

   (a) What is the conditional PDF $f_{W|C}(w)$?

   (b) What is the conditional expected value E[$W|C$]?

(c) Find the conditional variance, Var$[W|C]$.

5. **Functions of random variables:**

   (a) Find the PDF $f_Y(y)$ for the random variable Y, where $Y = X^3$ and X is a uniform random variable in the range (0,1).

   (b) Let X and Y be independent random variables with $f_X(x) = \exp(-x)u(x)$, and $f_Y(y) = \frac{1}{2}[u(y+1) - u(y-1)]$ and let $Z = X + Y$, where $u()$ denotes unit step function. What is the PDF of random variable Z? HINT:Use convolution!