THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring**

Problem Set 4

1. **Phase Locked Loop Demodulator (quickie):**

   (a) Using what you know about phase locked loops (PLLs), show that the PLL can be used as a FM demodulation circuit.

   SOLUTION: *Just use the $\dot{\phi}(t)$ output – it's the derivative of whatever was inside the argument of that incoming $\cos 2\pi f_c t + \omega(t)$. That is, $\dot{\phi}(t) = 2\pi f_c + \dot{\omega}(t)$. So you filter the D.C., and throw in a few multiplicative constants and you're done.*

   (b) How can you modify your PLL to obtain a PM demodulation circuit?

   SOLUTION: *Integrate the output of your demodulator above. But note, you've GOT to get rid of the DC offset, otherwise you'll have a lawsuit on your hands when $2\pi f_c t$ exceeds about ten kilovolts – how fast would that happen if the listener were tuned to 99.9FM? :)*

2. **Hilbert Transform Stuff for Culture:** The Hilbert-transform of a Fourier transformable signal $m(t)$, denoted by $m_H(t)$, is defined by

$$m_H(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(t)}{t-\tau}\, d\tau$$

In the frequency domain, we have

$$M_H(f) = -j\ sgn(f)M(f)$$

where $m(t)$, $M(f)$ and $m_H(t)$, $M_H(f)$ are Fourier transform pairs and $sgn(f)$ is the signum function. Using the definition of the Hilbert transform, show that a single-sideband modulated signal resulting from the message signal $m(t)$ and carrier $\cos(2\pi f_c t)$ of unit amplitude is given by

$$s(t) = \frac{1}{2}[m(t)\cos(2\pi f_c t) \pm m_H(t)\sin(2\pi f_c t)]$$

where the minus sign corresponds to the transmission of upper sideband and plus to the transmission of lower sideband.

SOLUTION: *Consider first the modulated signal*

$$s(t) = \frac{1}{2}[m(t)\cos(2\pi f_c t) \pm m_H(t)\sin(2\pi f_c t)]$$

Applying Fourier transform to the above equation, we get

$$S(f) = \frac{1}{4}[M(f-f_c) + M(f+f_c)] - \frac{1}{4j}[M_H(f-f_c) - M_H(f+f_c)]$$

From the defenition of Hilbert Transform, we have

$$M_H(f) = -j \; sgn(f)M(f)$$

where $sgn(f)$ *is the signum function. Equivalently, we may write*

$$\frac{-1}{j}M_H(f-f_c) = sgn(f-f_c)M(f-f_c)\frac{-1}{j}M_H(f+f_c) = sgn(f+f_c)M(f+f_c)$$

From the definition of signum function, we note the following for $f > 0$ *and* $f > f_c$

$$sgn(f-f_c) = sgn(f+f_c) = 1$$

Correspondingly, the equation for $S(f)$ *reduces to,*

$$S(f) = \frac{1}{4}[M(f-f_c) + M(f+f_c)] + \frac{1}{4}[M(f-f_c) - M(f+f_c)] = \frac{1}{2}M(f-f_c)$$

In words, the above result means that, except for a scaling factor, the spectrum of the modulated signal $s(t)$ *is the same as that of a DSB-SC modulated signal for* $f > f_c$*.*

For $f > 0$ *and* $f < f_c$*, we have*

$$sgn(f-f_c) = -1 sgn(f+f_c) = 1$$

Correspondingly, $S(f)$ *reduces to,*

$$S(f) = \frac{1}{4}[M(f-f_c) + M(f+f_c)] + \frac{1}{4}[-M(f-f_c) - M(f-f+c)] = 0$$

In words, for $f < f_c$*, the modulated signal* $s(t)$ *is zero.*

Combining the two results, the modulated signal $s(t)$ *represents a single sideband modulated signal containing only upper sideband. The result was derived for* $f > 0$*. This result also holds for* $f < 0$*, and can be easily proved in a similar manner using appropriate values for sgn function for various frequencies.*

3. **A Little Heterodyne Review:** Section 2.9 in your text describes the superheterodyne receiver. How precise do the RF filters (just after the antenna and after the mixer) need to be to extract a single FM radio station at carrier frequency 99.1MHz and no others just before the envelope detector of FIGURE 2.32 in your text? What is the primary benefit of using the superheterodyne structure?

   SOLUTION: *The channel separation in FM radio is 200KHz. This implies a single sided bandwidth of 100KHz for FM transmission though in practice 75KHz is the actual figure. In any case, to pull a single channel at 99.1MHz out of the air, we'd need a band pass filter with a bandwidth of 200KHz (single-sided) centered at 99.1MHz – that's 100KHz to either side of the carrier. The implied Q factor for such a filter would be about* $99.1 \times 10^6/10^5 \approx 1000$ *which is a very highly tuned filter/amplifier which is hard to make variable with a knob (potentiometer or variable capacitor). And you'd need one of these for each and every channel you wished to receive.*

   Alternately, you could try to take the signal directly down to baseband and do no filtering in the RF range. However, when these methods were being developed (and today still) it *was hard to get isolation between the high frequency local oscillator and both inputs of the mixer – so some local oscillator would leak into the mixer (along with the incoming desired signal) and you'd end up with large DC components as well as distortion in systems which used both the I and Q rail.*

So the answer is to build the wonderful filter (and amplifier) ONCE and do it at a frequency where the analog circuit design is not horrific, you don't worry about oscillator bleed through because it's relatively low frequency. However if there IS bleed through, it is D.C. and your signal is at IF and not D.C.. And most important from the multiplexing perspective, you don't get any image frequencies into your IF section.

So consider mixing the RF signal with a sinusoid at frequency f_Δ *where* $f_\Delta + f_{IF} = f_c$ *and* f_c *is the carrier frequency and* f_{IF} *is the intermediate frequency (***note:** *that this is the OPPOSITE of your book which does* $f_\Delta = f_c + f_{IF}$*). So, using our* $99.1MHz$ *carrier we want* $\pm f_\Delta = \pm(99.1MHz - f_{IF})$*. However, we note that a signal sitting at center frequency* $f = 99.1MHz - f_\Delta$ *if mixed with* f_Δ *will give you* $- \pm f_\Delta$ *which is spectrally the SAME as* $\pm f_\Delta$*. So, you want to make sure you filter out signals in the RF range that are within* $\pm f_\Delta$ *of the carrier.*

Using $10.7MHz$ *(the actual IF frequency used by FM radio) then means you need a filter with only a* Q *of about* $100MHz/10MHz = 10$ *in the RF front end to avoid polluting the desired signal at IF and this is a relatively easy task. Furthermore, in the IF stage, the Q instead of being* 1000 *is about* 100 *so you also have a design gain there.*

So that's the long and short of the heterodyne concept. These days however, there is a big push to do direct to baseband modulation since from a systems standpoint it's simpler. The driving force behind direct demodulation, *as it's called, is the advancing capability of integrated circuits, and in particular the availability of analog to digital converters which run at ridiculously high speeds (200 MHz sampling rates at 8 bits!) which enable you to sample the incoming signal above the nyquist rate and then perform all operations DIGITALLY!*

4. **Problem 2.40 in Haykin**

SOLUTION: *This is a neat problem it turns out. Notice that the input to the envelope detector is* $s(t) - s(t-T)$*. What does this difference remind of you (think small* T*). Yes, it reminds you of the definition of a derivative* $\frac{f(x+\Delta x) - f(x)}{\Delta x}$ *except that we don't divide through by* T*. And we know that we can decode FM by differentiating and then passing the result through an envelope detector! But, let's trudge through the long way first, just to be precise.*

In any case, it's easiest to work with the complex envelope so the input to the envelope detector is

$$\begin{aligned} s(t) - s(t-T) &= \Re\left\{e^{j\omega_c t + j\beta\sin\omega_m t} - e^{j\omega_c(t-T)+j\beta\sin\omega_m(t-T)}\right\} \\ &= \Re\left\{\underbrace{\left[e^{j\omega_c t + j\beta\sin\omega_m t}\right]}_{\textit{carrier}}\underbrace{\left[1 - e^{-j\omega_c T + j\beta(\sin\omega_m(t-T) - \sin\omega_m t)}\right]}_{\textit{envelope}}\right\} \end{aligned}$$

and we know that the envelope will pop out of the envelope detector, so let's concentrate on that.

We then use the identity $\sin(a+b) = \sin a\cos b + \sin b\cos a$ *to rewrite*

$$\sin\omega_m(t-T) = \sin\omega_m t\cos\omega_m T - \sin\omega_m T\cos\omega_m t \approx \sin\omega_m t - \omega_m T\cos\omega_m t$$

using the hints. So we now have

$$\left\{1 - e^{-j\omega_c T - j\beta\omega_m T\cos\omega_m t}\right\}$$

for the envelope term. Now, the hint implies that $|\omega_m T| \ll 1$ *and since both* β *and* $|\cos()|$ *are less than one, we have* $|\omega_m T\beta\cos\omega_m t| \ll 1$ *too. Therefore we can do a taylor expansion of the exponential to obtain*

$$\Re\left\{1 - e^{-j\omega_c T}(1 - j\beta\omega_m T\cos\omega_m t)\right\}$$

So, what we want to do is choose T *so that we get only the program material* $j\beta\cos\omega_m t$ *out of that envelope and* T *chosen such that* $\omega_c T = 2\pi$ *will do just fine so we're left with*

$$j\beta\omega_m T\cos\omega_m t$$

so that

$$s(t) - s(t-T) = \Re\left\{\underbrace{\left[e^{j\omega_c t + j\beta\sin\omega_m t}\right]}_{\textit{carrier}}\underbrace{\left[j\beta\omega_m T\cos\omega_m t\right]}_{\textit{envelope}}\right\}$$

the desired result for frequency demodulation.

5. **Cora the Advertising Maven:**

   WRUR, an FM radio station, has hired the famous Dr. Cora C. Enginoire to run advertising. Cora has decided to boost advertising revenue by playing the commercials louder!

   Assume that under normal circumstances WRUR has a carrier frequency of 98.7MHz and the output spectrum has significant components between 98.69MHz and 98.71MHz. The next nearest stations operate at 98.9MHz and 98.5MHz.

   By approximately what factor can Cora make the commercials louder before the neighboring stations complain to the FCC of radio interference. You must justify you answer.

   HINT: You may assume the following simplified form of Carson's rule: BW=$2\beta\omega_m$.

   SOLUTION: *Whatever* β *the station has been using before, it occupies a 10KHz band. Assuming the next nearest stations also occupy 10KHz band, there is 180KHz of "spare" band between stations. Cora can therefore make* β *18 times larger before she runs into interference problems. But the receiver does not compensate for larger* β *and simply sees a larger program material signal. Thus, the advertisements can be 18 times louder!*

   However, if the other stations follow a similar advertising strategy, they must all stay under 9 times louder to avoid interfering with each other.

6. **Evil in AM Systems:** A baseband signal $m(t)$ is used to produce a modulated signal $m(t)\cos 2\pi f_c t$. In the passband it is corrupted by two the addition of Evil signals so that the received signal is $r(t) = e_1(t)\cos 2\pi f_c t - e_2(t)\sin 2\pi f_c t + m(t)\cos 2\pi f_c t$. The Evil signals, $e_i(t)$, each have power density $N_0/2$ per Hz in double-sided band of width $2W$, and $m(t)$ has power P in the same band. (double-sided).

   (a) What is tht total power in each of the $e_i(t)$

   SOLUTION: $N_0 W$ *– just multiply the power density by the band in which it resides*

(b) Please design a coherent receiver to obtain $m(t)$ and calculate the signal to noise ratio at the output of your receiver.

HINT: You can do this the easy way or the hard way, depending on what you remember about synchronous demodulation.

SOLUTION: *For coherent detection, we already know that the sine and cosine rails are independent. So the only thing which makes it out of your synchronous receiver is* $m(t) + e_1(t)$. *The signal to noise ratio is then just* $\frac{P}{WN_0}$.

(c) Suppose you only have an envelope detector available and value of WN_0 is small relative P. What is the signal to noise at the output of your envelope detector.

HINT: Though you could probably copy a version of the answer from the text, you'll have more fun (and learn more) if you start from first principl es).

SOLUTION: *The instantaneous output power of envelop detector is*

$$\begin{aligned} y^2(t) &= (e_1(t) + m(t))^2 + e_2^2(t) \\ &= m^2(t) + 2e_1(t)m(t) + e_1^2(t) + e_2^2(t) \end{aligned}$$

Expected values: $E[m^2(t)] = P$, $E[e_i^2(t)] = N_0W$ *and* $E[m(t)e_1(t)] = 0$ *so that we have signal power* P *and noise power* $2N_0W$ *So, we can get* $SNR = \frac{P}{2WN_0}$ *or half the SNR of the coherent detector.*

You'll notice that this differs from the derivation in your book which explicitly considered large carrier AM.

7. **Cold Water Bucket at 3 AM:** QUICK! What is the effect on output signal to noise ratio of raising the modulation index for an FM signal. TICK, TICK, TICK, TICK, ... SPLASH!

SOLUTION: *Output signal to noise in an FM system goes up as the modulation index squared. So your SNR gets better FAST with increasing modulation index! That's one reason FM is sometimes called "poor man's spread spectrum."*