

Haykin: 4.1-4.4,5.1-5.7

1. Which of the following signals are orthogonal? You must show all work.
 - (a) $\cos t$ and $\sin t$ on $(0, \pi)$.
 - (b) $\cos t$ and $\sin t$ on $(0, \pi/2)$.
 - (c) t^2 and t^3 on $(-1, 1)$.
 - (d) $t \cos 2\pi t$ and $\cos 2\pi t$ on $(-\pi, \pi)$.
 - (e) $te^{-|t|}$ and $t^2e^{-t^2}$ on $(-1, 1)$.
2. Consider the signal $s(t)$ shown in Figure P4.1 (page 300, Haykin).
 - (a) Determine the impulse response of a filter matched to this signal and sketch it as a function of time.
 - (b) Plot the matched filter output as a function of time.
 - (c) What is the peak value of output.
3. Figure P4.2a (page 301, Haykin) shows a pair of pulses that are orthogonal to each other over the interval $[0, T]$. In this problem we investigate the use of this pulse pair to study a two-dimensional matched filter.
 - (a) Determine the matched filters for pulses $s_1(t)$ and $s_2(t)$ considered individually.
 - (b) Form a two dimensional matched filter by connecting two of the matched filters of part 1 in parallel, as shown in figure P4.2b (page 301, Haykin). Hence, demonstrate the following:
 - i. When the pulse $s_1(t)$ is applied to the two-dimensional matched filter, the response of the lower matched filter (sampled at time T) is zero.
 - ii. When the pulse $s_2(t)$ is applied to the two-dimensional matched filter, the response of the upper matched filter (sampled at time T) is zero.
 - (c) How would you design a matched filter bank for a set of n orthogonal pulses.
4. **Cora and the Tiger:** You are placed in a room with two doors and loudspeakers. You have been told that behind one door is untold wealth and behind the other, certain death. Of course, you are only told that any given door is equally likely to contain certain death or riches and that you MUST choose a door. Noise is played over the loudspeakers (really good speakers!) to confuse your thinking.

Luckily, your friend Cora the Communications Engineer has found a way to determine which door is which and together you've worked out a signaling system. If it's door 1 has riches, she'll whistle at some frequency ω_c . If door 2 has riches, she'll not whistle at all. Of course, Cora is outside the room, can only whistle with limited amplitude and the noise is loud.

Assume that you know the available signal is $A \cos(\omega_c t) + w(t)$ when Cora is whistling and just $w(t)$ otherwise. Assume that $w(t)$ is white, Gaussian, zero mean and has spectral height $N_0/2$.

- (a) If you can observe the signal for k cycles (on $[0, T = 2\pi k/\omega_c]$), what is the impulse response of the matched filter for the incoming signal $A \cos \omega_c t$? Please write your answer in analytic form.
 - (b) What is the expected value of the matched filter output at time T given the signal is absent? Show your work.
 - (c) What is the expected value of the matched filter output at time T given the signal is present? Show your work.
 - (d) What is the PDF of the matched filter output at time T given the signal is absent? Show your work.
 - (e) What is the PDF of the matched filter output at time T given the signal is present? Show your work.
 - (f) If your ears can act as a matched filter for the whistle-signal and you wish to be at least 99.999999% sure that you make the right choice of door, how few cycles k can you wait before making a choice? You may use the fact that $\frac{1}{2}\text{erfc}(\sqrt{11}) \approx 10^{-6}$. Show your work. NOTE: $\text{erfc}(x) = 2 \int_x^\infty e^{-z^2} dz$.
 - (g) Suppose Cora's a little drowsy and instead of whistling $\cos \omega_c t$ she whistles (unknownst to you) $\sin \omega_c t$. What is the probability that you choose the door with riches? Show your work.
5. Cora the communications engineer is the operator of an early warning sonar system. The system takes sonar input $f(t)$ and produces a number, Γ as

$$\Gamma = \int_0^2 f(t)c(t)dt$$

where $c(t)$ is some fixed signal. Notice that Γ is simply the correlation $R_{fc}(0)$ on the interval $(0, 2)$ for real signals $f(t)$ and $c(t)$.

Incoming enemy subs produce the signal $e(t) = \sin \pi t$. Incoming allied subs produce $a(t) = \cos \pi t$. Thus, $f(t)$ can be the sum of each, both or neither signal; i.e.,

- I no subs present $\rightarrow f(t) = 0$
- II enemy sub only $\rightarrow f(t) = e(t)$
- III allied sub only $\rightarrow f(t) = a(t)$
- IV both subs present $\rightarrow f(t) = a(t) + e(t)$

- (a) Which of the following signals should be chosen for $c(t)$ if Cora wishes to have Γ nonzero **ONLY** when an enemy submarine is present? Assume that both an enemy sub and an allied sub might be present at the same time. You must justify your answer quantitatively.
- $\cos \pi t$
 - $\cos(\pi t + \pi/4)$
 - $\cos(\pi t - \pi/4)$
 - $\sin 8\pi t$
 - $u(t) - 2u(t - 1) + u(t - 2)$
- (b) Suppose Cora wants to be able to distinguish events I through IV based on the value of Γ : Which $c(t)$ should she use? You must justify your answer quantitatively.