THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring 2006**

Final Exam

There are FIVE questions. You have the three hours to answer them. Read the WHOLE EXAM before doing the problems. Show all work. Answers given without work will receive no credit. GOOD LUCK!

1. *(50 points)* **Amplitude Modulation:** A signal $r(t) = m(t)(\cos 200000\pi t + \sin 200000\pi t)$ is sent over the air and captured by a receiver. You may assume the program material $m(t)$ is band limited to $\pm$1kHz and is always non-negative.

   (a) *(25 points)* Please sketch a block diagram for ANY receiver which will recover $m(t)$.

   (b) *(25 points)* Let
   $$c(t) = \sum_k (u(t-kT) - u(t-(k+1/2)T)$$
   and suppose we can synthesize $c(t-\tau)$ at our receiver for any $\tau \in \Re$ and $T \in \Re^+$. Please show how to build an envelope detector WITHOUT DIODES for detection of $m(t)$ using our generated signal $c(t-\tau)$ and a multiplier followed by a capacitor and resistor. A sketch would be most effective, but be sure to justify your answer.

2. *(50 points)* **Frequency Modulation:** Program material $m(t)$, bandlimited to $\pm W$, is transmitted using frequency modulation as in
   $$r(t) = A\cos\left(2\pi f_c t + \beta \int_0^t m(\tau) d\tau\right)$$
   where A is a constant, $f_c \gg W$ is the carrier frequency and β is a modulation index.

   (a) *(25 points)* Provide an explicit approximate analytic expression for $r(t)$ when it is assumed $|\beta| \ll 1$ (narrowband FM). Can $m(t)$ can be obtained via synchronous amplitude demodulation using $\cos 2\pi f_c t$? Why/why not?

   (b) *(25 points)* Provide a carefully labeled block diagram of a phase locked loop FM receiver which extracts $m(t)$ from $r(t)$. Your diagram should include a multiplier, a low pass filter, a scalar multiplication (amplifier) and a voltage controlled oscillator. Should the amplifier be an inverting or a non-inverting amplifier? What is the minimum bandwidth of the low pass filter? You must justify your answers.

3. *(50 points)* **Sampling Theory, Signal Space and Parseval:** A signal $m(t)$ bandlimited to $\pm W$ can be represented by its samples $m(kT)$ where $T \le \frac{1}{2W}$. This so-called Nyquist Sampling Theorem is most easily proven by considering impulsive sampling $\sum_k m(kT)\delta(t-kT)$ and applying this signal to an ideal low pass filter with bandwidth $\pm W$ to recover $m(t)$.

(a) *(10 points)* Show that an ideal low pass filter with bandwidth $\pm W$ has an impulse response **proportional to**

$$h(t) = \frac{1}{\sqrt{2W}} \frac{\sin 2\pi W t}{\pi t}$$

(b) *(20 points)* Please provide an expression for $m(t)$ in terms of its samples $\{m(kT)\}$ and $h(t)$.

(c) *(20 points)* Please verify that the $\{h(t-kT)\}$ are orthonormal waveforms. You might find Parseval's relation useful:

$$\int_{-\infty}^{\infty} x(t)y(t)dt = \int_{-\infty}^{\infty} X(f)Y^*(f)df$$

where $x(t)$ and $y(t)$ are real signals and $X(f)$ and $Y(f)$ are their respective Fourier transforms.

4. *(50 points)* **Digital Detection:** You are given two signals on the interval $(-1,1)$: $s_0(t) = u(t+1) - u(t-1)$ and $s_1(t) = t$. These signals are to be used to send information bits with $s_0(t)$ corresponding to a zero and $s_1(t)$ to a one. Thus

$$r(t) = s_i(t) + n(t)$$

where $i = \{0,1\}$ and $n(t)$ is zero mean white Gaussian noise with spectral height σ^2.

(a) *(25 points)* Find a signal space with two orthonormal basis functions $\phi_1(t)$ and $\phi_2(t)$ in which both $s_1(t)$ and $s_0(t)$ can be represented. Label the points in your signals space which correspond to $s_1(t)$ and $s_0(t)$.

(b) *(25 points)* Please provide a block diagram for a minimum probability of error receiver assuming that sending a one or zero is equally likely. Sketch the decision region of your receiver in the signal space.

5. *(50 points)* **Cora Does a Double Take:** Cora the communications engineer has been challenged to a detection duel by her arch nemisis, Marty the Squirrel. Marty gets to choose a symbol b and transmit it. Cora has access to one of two received signals $r_1(t)$ and $r_2(t)$ where

$$r_1(t) = bs(t) + n(t)$$
$$r_2(t) = 0.1bs(t) + n(t)$$

The signal $s(t)$ is a unit energy signature waveform, $b = \pm 1$ equiprobably and $n(t)$ is a zero mean Gaussian white noise signal with spectral height $N_0/2$. Cora has to guess whether -1 or $+1$ was sent. If she guesses right, Cora gets to use her flamethrower on Marty's bushy tail. If she guesses wrong, Marty gets to tap dance on her face with his hobnail squirrel boots.

(a) *(25 points)* Assuming matched filter detection, if Cora has a choice of using either $r_1(t)$ or $r_2(t)$, which should she use to minimize her chances of facial dents? You must justify your answer.

(b) *(25 points)* Now suppose Cora has access to both $r_1(t)$ and $r_2(t)$. Please provide a block diagram of a receiver which maximizes the probability of a flaming tail. What is this probability?