THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring 2005**

Final Exam

There are FIVE questions. You have the three hours to answer them. Read the WHOLE EXAM before doing the problems. Show all work. Answers given without work will receive no credit. GOOD LUCK!

1. *(50 points)* **Continuous Modulation:** A signal $r(t) = b(t)\cos 2000\pi t$ is received where

$$b(t) = \sum_{k=-\infty}^{\infty} b_k p(t-kT)$$

with $p(t) = \frac{\sin \pi t}{\pi t}$ and the b_k having arbitrary real values.

   (a) *(25 points)* Can $b(t)$ be recovered exactly using a synchronous AM demodulator? If so, why and how? If not, why not?

   (b) *(25 points)* Suppose now that $p(t) = u(t) - u(t-1)$. Can $b(t)$ be recovered exactly using a synchronous AM demodulator? If so, why and how? If not, why not?

2. *(50 points)* **Cora, Marty and DaMar:** Cora the communications engineer has a dog named Data. As you know by now, Cora and Marty are not exactly friendly. So it was with great dismay that Cora watched Data gave birth to DaMar, the squirrel-dog.

DaMar is much more squirrel than dog and roams the nearby forest foraging for nuts just like Marty. Cora, seeking revenge, wants to drive Marty from the forest and she's decided to destroy Marty's nut caches. However, she has an auntly soft spot for DaMar and does not want to destroy his caches.

Now, DaMar is a rather large squirrel and collects more nuts on average than Marty does. In fact the number of nuts N in a given cache is

$$f_N(n|\text{DaMar}) = \frac{(\lambda_D)^n}{n!}e^{-\lambda_D}$$

for DaMar and

$$f_N(n|\text{Marty}) = \frac{(\lambda_M)^n}{n!}e^{-\lambda_M}$$

for Marty where $\lambda_D = \alpha\lambda_M$ and $\alpha > 1$

When Cora comes across a nut cache, she counts the number of nuts N and makes a decision about whether to destroy the cache. Your job it to help her devise an appropriate decision rule. You may assume that Cora comes across a cache belonging to Marty or one belonging to DaMar with equal probability.

(a) *(20 points)* Based on n the number of nuts in the cache, please provide a decision rule which minimizes the probability that Cora makes a mistake. A mistake is defined as when Cora destroys a cache belonging to DaMar or leaves a cache belonging to Marty.

(b) *(20 points)* For $\alpha = e^2$ and $\lambda_M = 1$, please provide an analytic expression for the probability that Cora makes a mistake using your decision rule. (NOTE: $e^2 \approx 7.39$.)

What happens to this probability if $\alpha = e^4$. (NOTE: $e^4 \approx 54.6$.)

(c) *(10 points)* Suppose Cora is only worried about the probability that she destroys one of DaMar's caches and does not worry at all about erroneously leaving one of Marty's. What is her optimal policy in that case? What is the probability she mistakenly destroys one of DaMar's caches?

3. *(50 points)* **Digital Modulation:**

An information signal $b(t)$ has the form

$$b(t) = \sum_{k=-\infty}^{\infty} b_k p(t - kT)$$

where $p(t) = u(t) - u(t-T)$ (i.e., $p(t)$ is a unit-height pulse on $(0,T]$), the $b_k \in \{0,1\}$ are equiprobable random information digits and $T > 0$. Assume that the receiver knows the structure of the signal $b(t)$ but not the specific $\{b_k\}$.

(a) *(15 points)* If the received signal $r(t) = b(t)\cos\frac{2\pi}{T}t$, can you devise a receiver which recovers $b(t)$ EXACTLY if you have a copy of $\cos\frac{2\pi}{T}t$ available at the receiver. If so, how? If not, why not?

HINT: Try sketching an example of $r(t)$.

(b) *(15 points)*

Now suppose you only have some sinusoid $\cos(\frac{2\pi}{T}t + \phi)$ available at the receiver where ϕ is some phase offset. Can you still always recover $b(t)$ exactly from $r(t) = b(t)\cos\frac{2\pi}{T}t$? If so, how? If not, why not?

(c) *(20 points)* Now suppose zero mean white Gaussian noise $n(t)$ with spectral height N_0 corrupts the received signal so that $r(t) = b(t)\cos\frac{2\pi}{T}t + n(t)$. Please carefully sketch and label a receiver which recovers the $\{b_k\}$ with minimum probability of error. You must also include the rule used for the "decision box". State all assumptions.

4. *(50 points)* **Quantization:** You are given the 2-bit quantizer shown in FIGURE 1 and are told that it's an optimal quantizer for a signal which has uniform probability density between zero and one. Is the quantizer optimal?

(a) *(10 points)* Why is $Q(x)$ called a 2-bit quantizer?

(b) *(40 points)* Is the quantizer optimal? If so, why? If not why not – and what IS the optimal quantizer?

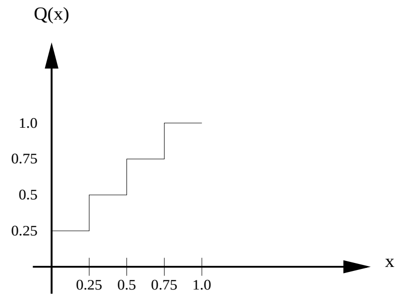

Figure 1: Quantizer for Problem 4

5. *(50 points)* **Signal Space:** You are given two signals on the interval $(0,1)$: $\phi_1(t) = u(t) - u(t-1)$ and $\phi_2(t) = u(t) - 2u(t-0.5) + u(t-1)$.

   (a) *(20 points)* Show that the $\phi_i(t)$ are orthonormal.

   (b) *(30 points)* Provide an analytic expression for each of the signal points depicted in FIGURE 2. You may also sketch them if you like.

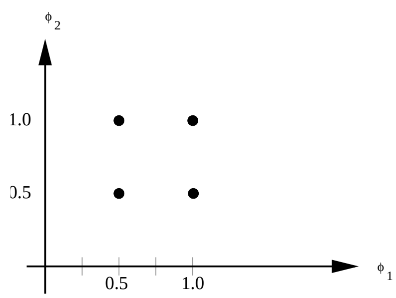

Figure 2: Signal points for Problem 5