THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring 2004**

Final Examination

There are FIVE questions. You have three hours to answer them. Show all work. Answers given without work will receive no credit. GOOD LUCK!

1. *(35 points)* **Continuous Modulation:** For each of the following parts, $m(t)$ is analog program material, band limited to $\pm W$ Hertz with $|m(t)| \leq 1$. The carrier frequency is $\omega_c \gg 2\pi W$ and the signal at the receiver is $r(t)$.

   (a) *(15 points)* If $r(t) = m(t)\cos\omega_c t$, carefully sketch a block diagram for a receiver which can successfully recover $m(t)$.

   (b) *(10 points)* You are given a wideband FM signal $r(t) = \cos\left(\omega_c t + k_f \int_0^t m(\tau)d\tau\right)$. Assuming $0 < k_f < \omega_c$, carefully sketch a block diagram for a receiver which has as one of its elements an envelope detector.

   (c) *(10 points)* If $m(t) = \cos 2000\pi t$ and $k_f = 20$, what is the approximate bandwidth of the modulated signal $r(t)$.

2. *(30 points)* **Quantization:** A signal has amplitude PDF $f_X(x)$ and we wish to design a quantizer $Q(x)$ with N levels which minimizes the mean square error between X and $Q(x)$.

   PROVE: if $f_X(x)$ is uniform on $\pm A$ where A is some constant, equal sized steps between vertical levels is optimal.

   HINT: The solution to the homogeneous difference equation $y(k+2) - 2y(k+1) + y(k) = 0$ is $\alpha + k\beta$ where α and β are constants.

3. *(50 points)* **Stationarity and Gaussian Processes:**

   (a) *(10 points)* Formally define Strict Sense Stationarity (SSS) for a random process, $X(t)$.

   (b) *(10 points)* Formally define Wide Sense Stationarity (WSS) for a random process, $X(t)$.

   (c) *(15 points)* A zero mean white Gaussian process $X(t)$ with spectral height N_0 is integrated to obtain $Z(t) = \int_0^t X(\tau)d\tau$, $t \geq 0$. What is the mean of $Z(t)$? What is its correlation? Is $Z(t)$ stationary? Why/why not?

   (d) *(10 points)* What is the average power of the process on the interval $(0, T)$ where the average power of a signal $x(t)$ is defined as

$$\bar{P} = \frac{1}{T}\int_0^T x^2(t)dt$$

   (e) *(5 points)* Is $Z(t)$ in the previous part a Gaussian process? Why/why not?

4. *(60 points)* **Cora and the Alien Invasion:** Cora the Communications Engineer has been hired by NASA to investigate the possibility of an imminent alien invasion through measurements taken by the Voyager deep space probe as it pierces the heliosphere (extended solar neighborhood). Through top secret research, NASA has determined that the signal level S follows the following distributions:

$$f_{S|H_0}(s|H_0) = \lambda^2 s e^{-\lambda s}$$

and

$$f_{S|H_1}(s|H_1) = \lambda s e^{-\frac{1}{2}\lambda s^2}$$

where H_1 means the aliens are planning an invasion, and H_0 not. In both cases, $s \geq 0$. Your job is to help Cora design a decision box which takes the measurement S and produces a decision about whether the aliens are invading or not and does so with minimum probability of error.

   (a) *(20 points)* Please sketch the two conditional distributions for $\lambda = 1$.

   (b) *(20 points)* If the aliens are planning an invasion with probability p, please provide an appropriate likelihood ratio for the decision regions associated with H_0 and H_1.

   (c) *(20 points)* Please determine analytic expressions for decision regions for $p = 0.5$. What does your region reduce to if $\lambda = 1$?

5. *(40 points)* **Signal Space:** A two-dimensional signal space uses basis functions $\phi_1(t) = 1$ and $\phi_2(t) = \sqrt{2}\cos 2\pi t$ on an interval $(0,1)$.

   (a) *(10 points)* Please provide a signal space vector representation $\mathbf{s}_k$ for each of the functions $s_1(t) = \cos^2 \pi t$ and $\sin^2 \pi t$ so that

$$s_k(t) = \mathbf{s}_k^\top \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix} = s_{k1}\phi_1(t) + s_{k2}\phi_2(t)$$

   What is the energy in each signal? What is the distance between these two points in signal space?

   (b) *(10 points)* On an interval $(0,1)$ one of these signals is sent with equal probability. Zero mean white Gaussian noise of spectral height N_0, $w(t)$, is added to the transmission and a minimum probability of error receiver is used to decode whether signal 1 or signal 2 was sent. Under this scenario, the probability of error is some number P_e

   Now, suppose we change the signal design and keep $s_1(t)$ as is but change $s_2(t) = -s_1(t)$. Does the total signal energy change? Does the probability of error go up or down? Why?

   (c) *(20 points)* Plot the two pairs of signal points in the same signal space coordinate frame and comment on binary signal design (where you have only two possible signals) under signal energy constraints in a multidimensional signal space. You may guess (with verbal justification) but to receive full credit you must PROVE your assertion.