School of Engineering
Department of Electrical and Computer Engineering

332:221 | **Principles of Electrical Engineering I** | **Fall 2012**

Quiz 2

No calculators, no books, no class notes, no nuttin'! Just a pencil/pen, your one side of 8.5×11 ***cheat sheet and you. Final answers must appear in the appropriate box. Show your work outside the box.***

1. *(24 pts)* **Really Basic Stuff:**

   (a) *(2 pts)* The current I_R through a resistor with voltage V_R is $I_R = \boxed{\frac{V_R}{R}}$

   (b) *(2 pts)* The current I_C through a capacitor with voltage V_C is $I_c = \boxed{C\frac{dV_c}{dt}}$

   (c) *(2 pts)* The voltage across an inductor with current I_L is $V_L = \boxed{L\frac{dI_L}{dt}}$

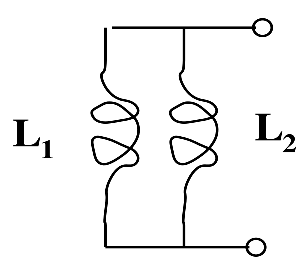
   (d) *(2 pts)* At radian frequency $\omega = 10$ what is the impedance of a capacitor of value C?

   $\boxed{\frac{0.1}{Cj}}$

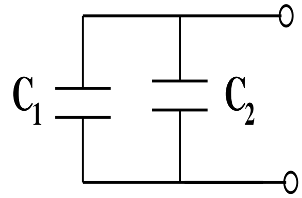
   (e) *(2 pts)* At radian frequency $\omega = 0.1$ what is the impedance of an inductor of value L?

   $\boxed{0.1Lj}$

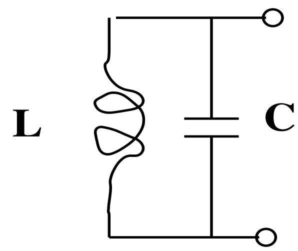
   (f) *(2 pts)* At radian frequency ω, $Z_{\text{eq}} = \boxed{\frac{L_1L_2}{L_1+L_2}j\omega}$



   (g) *(2 pts)* At radian frequency ω, $Z_{\text{eq}} = \boxed{\frac{1}{(C_1+C_2)j\omega}}$



(h) *(2 pts)* At radian frequency ω, $Z_{\text{eq}} =$ $\boxed{\frac{\frac{1}{Cj\omega}Lj\omega}{Lj\omega+\frac{1}{Cj\omega}} = \frac{Lj\omega}{LC(j\omega)^2+1}}$



(i) *(2 pts)* Write j in (exponential) phasor notation $\boxed{1 \cdot e^{j\pi/2}}$

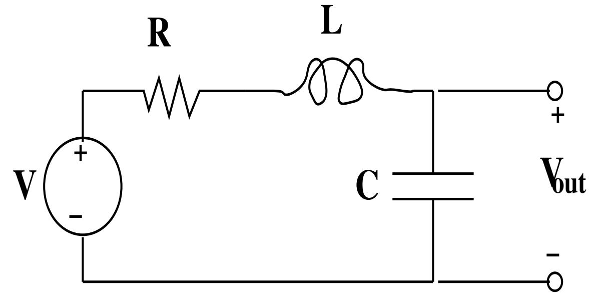
(j) *(2 pts)* Write $e^{j\pi/2}$ in $a + bj$ notation $\boxed{j}$

(k) *(2 pts)* $X = \frac{a+bj}{c+dj}$. What is $\Re\{X\}$? $\boxed{\frac{ac+bd}{c^2+d^2}}$

(l) *(2 pts)* What is $e^{-j\pi} + e^{j\pi/2}$ in (exponential) phasor notation?

$$\boxed{-1 + j = \sqrt{2}e^{j3\pi/4}}$$

2. *(12 pts)* **Less Basic:**



(a) *(6 pts)* Please derive the transfer function from phasor $\mathbf{V}$ to phasor $\mathbf{V}_{\text{out}}$.

$$H(j\omega) = \frac{1}{1+RCj\omega+LC(j\omega)^2}$$

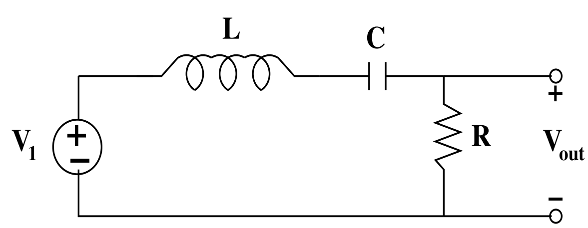
(b) *(6 pts)* The transfer function between phasor $\mathbf{V}$ and the phasor voltage across the resistor is

$$H(j\omega) = \frac{RCj\omega}{1 + RCj\omega + LC(j\omega)^2}$$

Please write down the differential equation relating the input voltage $V(t)$ to the voltage across the resistor, $V_R(t)$.

$$RC\frac{dV}{dt} = V_{\textbf{\textit{out}}}(t) + RC\frac{dV_{\textbf{\textit{out}}}}{dt} + LC\frac{d^2V_{\textbf{\textit{out}}}}{dt}$$

3. *(14 pts)* **Thevenin/Norton:**

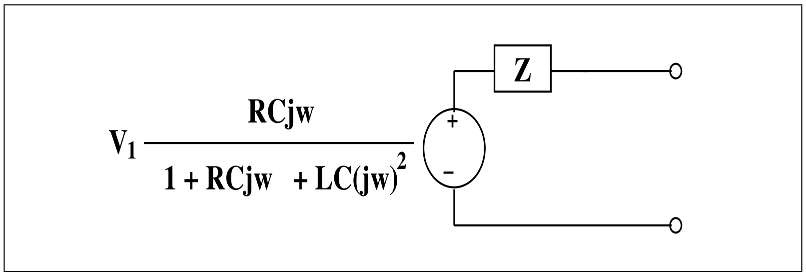


We assume sinusoidal steady state at frequency ω and input phasor $\mathbf{V}_1$.

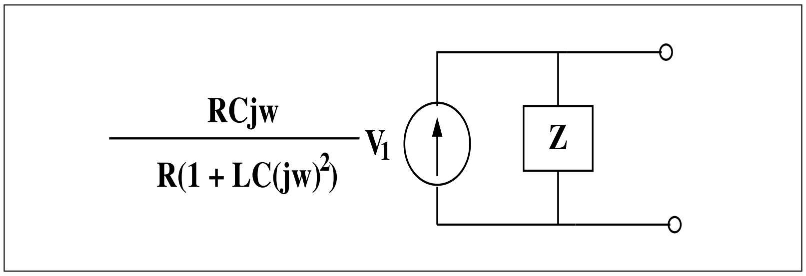
(a) *(6 pts)* What is the input impedance of the depicted circuit?

$$Z_{\textbf{eq}} = \frac{R(LC(j\omega)^2+1)}{RCj\omega+LC(j\omega)^2+1}$$

(b) *(4 pts))* Sketch and label the Thevenin equivalent as seen from V_{out}.



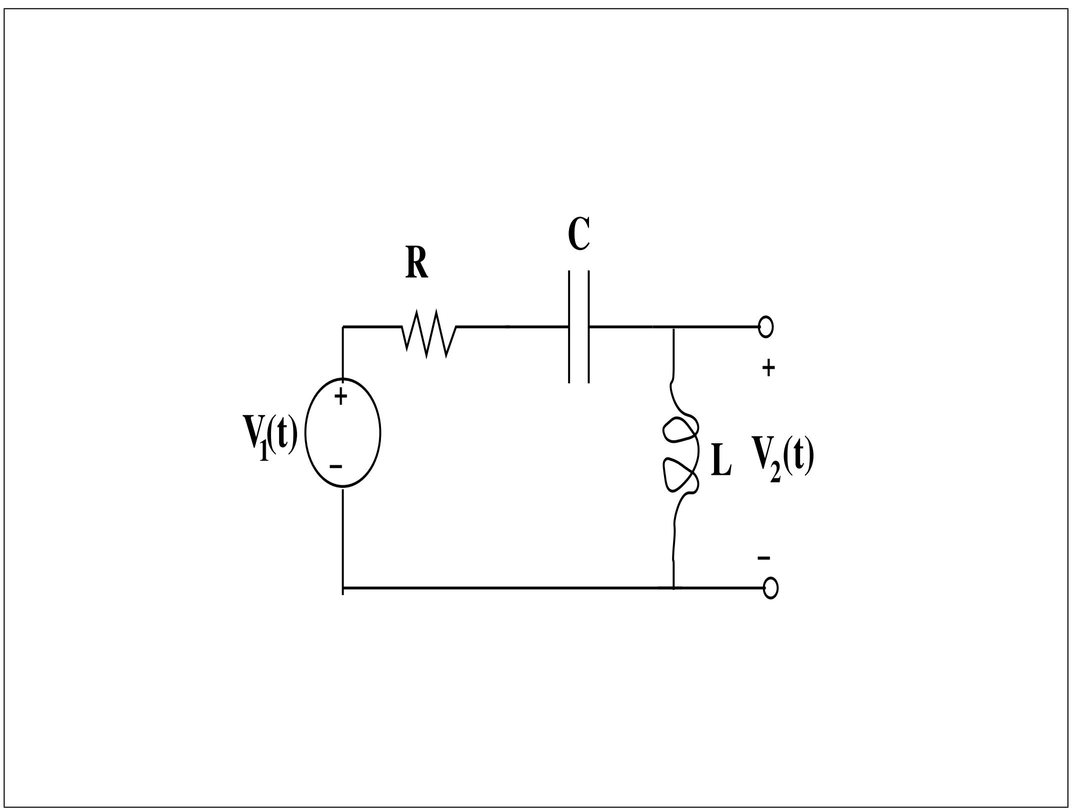
(c) *(4 pts))* Sketch and label the Norton equivalent as seen from V_{out}.



4. *(20 pts)* **Transfer Function Inferences:**

$$H(j\omega) = \frac{LC(j\omega)^2}{1 + RCj\omega + LC(j\omega)^2}$$

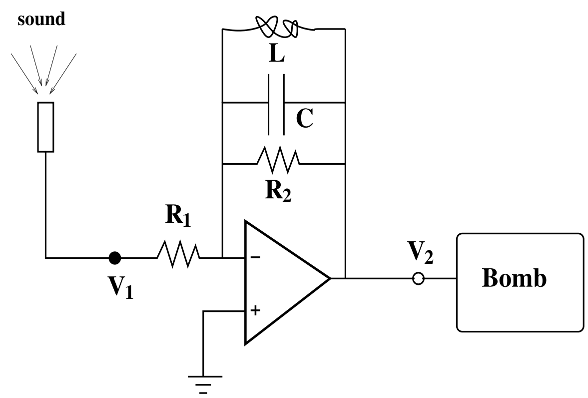
(a) *(10 pts)* Draw a circuit with transfer function $H(j\omega)$



(b) *(10 pts)* Suppose $LC = 1$ and $RC = 1$. If input $\cos \omega t$ produces $-\sin \omega t$ as the output, what is ω?

$\cos \to -\sin$ **is a** $\pi/2$ **phase shift and implies** $H(j\omega) = j$**.** $H(j\omega) = -\omega^2/(1 + j\omega - \omega^2) = j$**.** $\omega = 1$ **works.**

5. *(30 pts)* **Emma and Castor Whistling in the Dark:**



Emma the Electrical Engineer again finds herself battling the intelligent beaver, Dr. Castor Canadensis. She knows that Castor likes to go walking in the forest and becomes uneasy as he waddles past the Beaver Burial Grounds (BBG). When Castor is uneasy, he starts whisling through his buck teeth at the annoyingly high frequency of $f = 1000Hz$. Emma decides to set a trap using the circuit shown.

As shown in her schematic, the output of a microphone is fed into an inverting amplifier with components as shown. Castor whistles in a rather boring way – at constant amplitude 10mV. Thus, if Castor is whistling, the input to the amplifier is $V_1 = 0.01 \cos(2000\pi t + \phi_c)$ where ϕ_c is an arbitrary phase angle. The output of Emma's circuit, V_2, triggers a paintball bomb when its amplitude exceeds one volt.

However, Emma does not want her trap springing if it's not Castor who's walking by the BBG. Specifically, her boss also takes walks in the forest and whistles all the time. Emma's boss is a HUGE man with HUGE lips and GIGANTIC cheeks, so his (also rather monotonous) whistle is MUCH lower in frequency: $V_1 = 0.01 \cos(200\pi t + \phi_b)$ where ϕ_b is another arbitrary phase angle. Needless to say, if the paintball bomb goes off when her boss is passing the BBG, Emma will be fired.

(a) *(15 pts)* Assuming sinusoidal inputs at radian frequency ω, what is the transfer function from V_1 to V_2?

$$H(j\omega) = -\frac{R_2 L j\omega}{R_1(R_2 + Lj\omega + R_2 LC(j\omega)^2)}$$

(b) *(5 pts)*
Assume: $C = \frac{0.0005}{\pi}F$, $R_1 = 1k$, $L = \frac{0.0005}{\pi}H$, $R_2 = 200k$. At what frequency ω (if any) is V_1 in phase with V_2?

In phase when** $H(j\omega)$ **is real. This happens when** $1+(j\omega)^2LC = 0$. **Thus,** $\omega = \sqrt{1/LC}$ **or

$$f = \frac{1}{2\pi}\sqrt{\frac{\pi^2}{5\times10^{-4}5\times10^{-4}}} = \frac{1}{2\pi}\frac{\pi}{5\times10^{-4}} = 1000Hz$$

(c) *(5 pts)*
Assume: $C = \frac{0.0005}{\pi}F$, $R_1 = 1k$, $L = \frac{0.0005}{\pi}H$, $R_2 = 200k$. Will Emma's trap work when Castor goes past the BBG? Why/Why not?

Yes. The transfer function value at** $f = 1000Hz$ **(**$\omega = 2000\pi$**) is** $R_2/R_1 = 200$ **so that the output voltage amplitude is** $200 \times 10mV = 2V$ **– which is enough to trip the bomb.

HINT: $|A/B| = |A|/|B|$.

(d) *(5 pts)* Assume: $C = \frac{0.0005}{\pi}F$, $R_1 = 1k$, $L = \frac{0.0005}{\pi}H$, $R_2 = 200k$. Suppose Emma's boss passes the BBG before Castor does. Will Emma be fired? Why/Why not?

No Emma will not be fired. The transfer function magnitude at** $f = 100Hz$ **(**$\omega = 200\pi$**) is

$$\frac{R_2}{R_1}\frac{L\omega}{\sqrt{R_2^2(1-\omega^2LC)^2 + L^2\omega^2}} = 200 \times \frac{0.1}{\sqrt{4\times10^{10}(1-0.01)^2 + 0.01}}$$

which is clearly MUCH smaller than the requisite 100 needed to trip the bomb with a 10mv input signal

HINT: $|A/B| = |A|/|B|$.