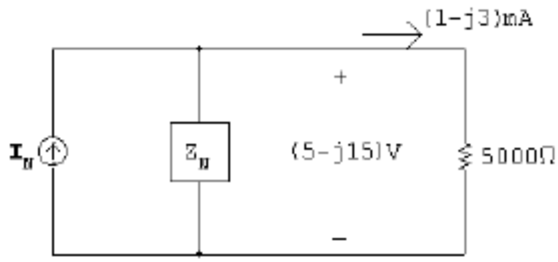
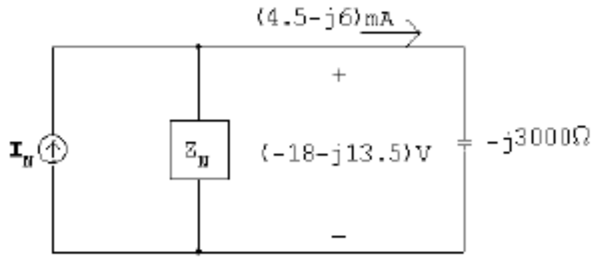
ECE221 Problem Set 9 Solutions (9.43, 9.48, 9.56, 9.59, 9.62, 9.64, 9.71, 9.84, 9.87)

P 9.43



$$\mathbf{I}_N = \frac{5-j15}{Z_N} + (1-j3)\,\text{mA}, \quad Z_N \text{ in k}\Omega$$

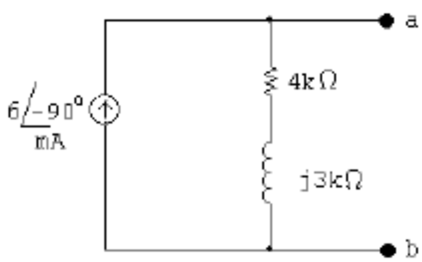


$$\mathbf{I}_N = \frac{-18-j13.5}{Z_N} + 4.5 - j6\,\text{mA}, \quad Z_N \text{ in k}\Omega$$

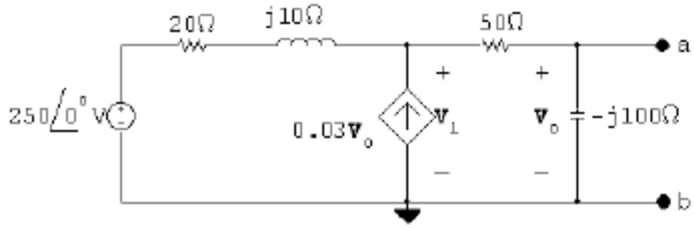
$$\frac{5-j15}{Z_N} + 1 - j3 = \frac{-18-j13.5}{Z_N} + (4.5-j6)$$

$$\frac{23-j15}{Z_N} = 3.5 - j3 \quad \therefore \quad Z_N = 4 + j3\,\text{k}\Omega$$

$$\mathbf{I}_N = \frac{5-j15}{4+j3} + 1 - j3 = -j6\,\text{mA}$$



P 9.48 Open circuit voltage:



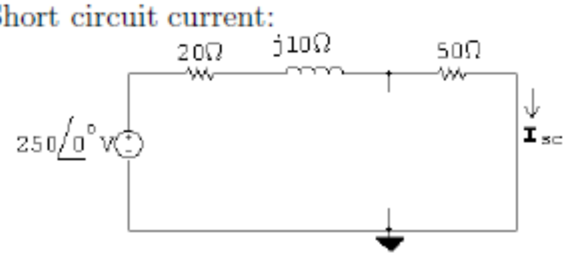
$$\frac{\mathbf{V}_1 - 250}{20 + j10} - 0.03\mathbf{V}_o + \frac{\mathbf{V}_1}{50 - j100} = 0$$

$$\therefore \quad \mathbf{V}_o = \frac{-j100}{50 - j100}\mathbf{V}_1$$

$$\frac{\mathbf{V}_1}{20 + j10} + \frac{j3\mathbf{V}_1}{50 - j100} + \frac{\mathbf{V}_1}{50 - j100} = \frac{250}{20 + j10}$$

$$\mathbf{V}_1 = 500 - j250\,\text{V}; \qquad \mathbf{V}_o = 300 - j400\,\text{V} = \mathbf{V}_{\text{Th}}$$

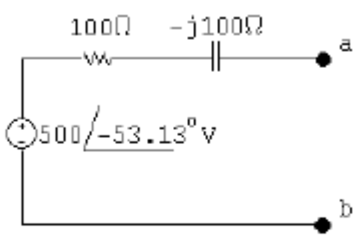
Short circuit current:



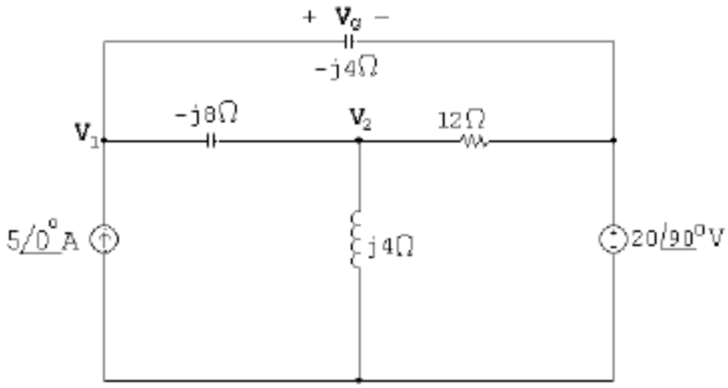
$$\mathbf{I}_{\text{sc}} = \frac{250\underline{/0^\circ}}{70 + j10} = 3.5 - j0.5\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{300 - j400}{3.5 - j0.5} = 100 - j100\,\Omega$$

The Thévenin equivalent circuit:



P 9.56 Set up the frequency domain circuit to use the node voltage method:



At $\mathbf{V}_1$: $$-5\underline{/0^\circ} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - 20\underline{/90^\circ}}{-j4} = 0$$

At $\mathbf{V}_2$: $$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - 20\underline{/90^\circ}}{12} = 0$$

In standard form:

$$\mathbf{V}_1\left(\frac{1}{-j8} + \frac{1}{-j4}\right) + \mathbf{V}_2\left(-\frac{1}{-j8}\right) = 5\underline{/0^\circ} + \frac{20\underline{/90^\circ}}{-j4}$$

$$\mathbf{V}_1\left(-\frac{1}{-j8}\right) + \mathbf{V}_2\left(\frac{1}{-j8} + \frac{1}{j4} + \frac{1}{12}\right) = \frac{20\underline{/90^\circ}}{12}$$

Solving on a calculator:

$$\mathbf{V}_1 = -\frac{8}{3} + j\frac{4}{3} \qquad \mathbf{V}_2 = -8 + j4$$

Thus

$$\mathbf{V}_g = \mathbf{V}_1 - 20\underline{/90^\circ} = -\frac{8}{3} - j\frac{56}{3}\,\mathrm{V}$$

P 9.59 Write a KCL equation at the top node:

$$\frac{\mathbf{V}_o}{-j8} + \frac{\mathbf{V}_o - 2.4\mathbf{I}_\Delta}{j4} + \frac{\mathbf{V}_o}{5} - (10 + j10) = 0$$

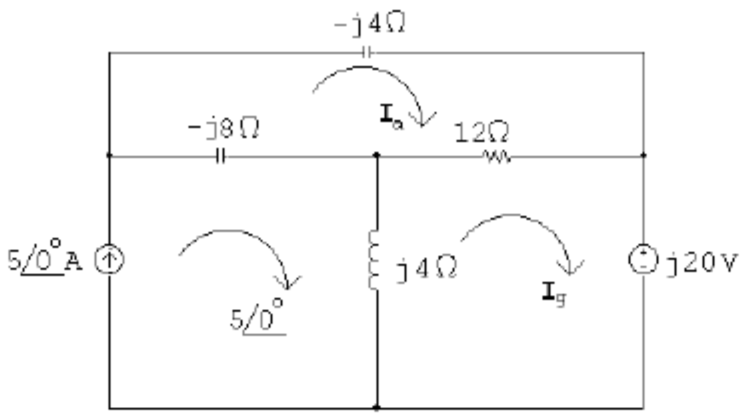
The constraint equation is:

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j8}$$

Solving,

$$\mathbf{V}_o = j80 = 80\underline{/90^\circ}\text{ V}$$

P 9.62



$$(12 - j12)\mathbf{I}_a - 12\mathbf{I}_g - 5(-j8) = 0$$

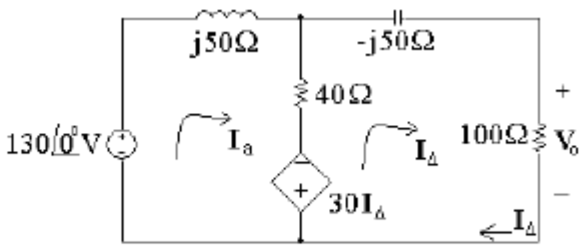
$$-12\mathbf{I}_a + (12 + j4)\mathbf{I}_g + j20 - 5(j4) = 0$$

Solving,

$$\mathbf{I}_g = 4 - j2 = 4.47\underline{/-26.57^\circ}\text{ A}$$

P 9.64 $j\omega L = j10{,}000(5 \times 10^{-3}) = j50\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(10{,}000)(2 \times 10^{-6})} = -j50\,\Omega$$



$$130\underline{/0^\circ} = (40 + j50)\mathbf{I}_a - 40\mathbf{I}_\Delta + 30\mathbf{I}_\Delta$$

$$0 = -40\mathbf{I}_a + 30\mathbf{I}_\Delta + (140 - j50)\mathbf{I}_\Delta$$

Solving,

$$\mathbf{I}_\Delta = (400 - j400)\,\text{mA}$$

$$\mathbf{V}_o = 100\mathbf{I}_\Delta = 40 - j40 = 56.57\underline{/-45^\circ}$$

$$v_o = 56.57\cos(10{,}000t - 45^\circ)\,\text{V}$$

P 9.71 **[a]** $\dfrac{1}{j\omega C} = \dfrac{-j10^9}{(10^6)(10)} = -j100\,\Omega$

$$\mathbf{V}_g = 30\underline{/0^\circ}\,\text{V}$$

$$\mathbf{V}_\text{p} = \frac{\mathbf{V}_g(1/j\omega C_o)}{25 + (1/j\omega C_o)} = \frac{30\underline{/0^\circ}}{1 + j25\omega C_o} = \mathbf{V}_\text{n}$$

$$\frac{\mathbf{V}_\text{n}}{100} + \frac{\mathbf{V}_\text{n} - \mathbf{V}_o}{-j100} = 0$$

$$\mathbf{V}_o = \frac{1 + j1}{j}\mathbf{V}_\text{n} = (1 - j1)\mathbf{V}_\text{n} = \frac{30(1 - j1)}{1 + j25\omega C_o}$$

$$|\mathbf{V}_o| = \frac{30\sqrt{2}}{\sqrt{1 + 625\omega^2 C_o^2}} = 6$$

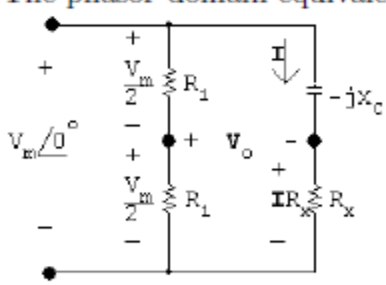
Solving,

$$C_o = 280\,\text{nF}$$

[b] $\mathbf{V}_o = \dfrac{30(1 - j1)}{1 + j7} = 6\underline{/-126.87^\circ}$

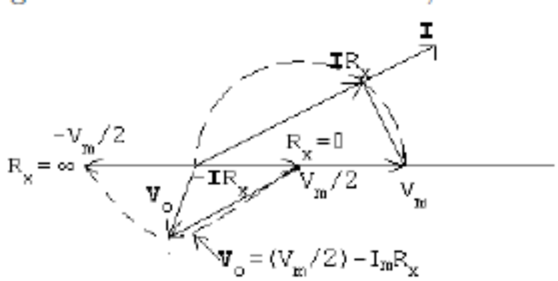
$$v_o = 6\cos(10^6 t - 126.87^\circ)\,\text{V}$$

P 9.84 The phasor domain equivalent circuit is



$$V_o = \frac{V_m}{2} - \mathbf{I}R_x; \qquad \mathbf{I} = \frac{V_m}{R_x - jX_C}$$

As R_x varies from 0 to ∞, the amplitude of v_o remains constant and its phase angle increases from 0° to -180°, as shown in the following phasor diagram:



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P 9.87 [a] $\mathbf{I}_1 = \frac{120}{24} + \frac{240}{8.4 + j6.3} = 23.29 - j13.71 = 27.02\underline{/-30.5^\circ}\text{ A}$

$$\mathbf{I}_2 = \frac{120}{12} - \frac{120}{24} = 5\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_3 = \frac{120}{12} + \frac{240}{8.4 + j6.3} = 28.29 - j13.71 = 31.44\underline{/-25.87^\circ}\text{ A}$$

$$\mathbf{I}_4 = \frac{120}{24} = 5\underline{/0^\circ}\text{ A}; \qquad \mathbf{I}_5 = \frac{120}{12} = 10\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_6 = \frac{240}{8.4 + j6.3} = 18.29 - j13.71 = 22.86\underline{/-36.87^\circ}\text{ A}$$

[b] When fuse A is interrupted,

$\mathbf{I}_1 = 0$	$\mathbf{I}_3 = 15\text{ A}$	$\mathbf{I}_5 = 10\text{ A}$
$\mathbf{I}_2 = 10 + 5 = 15\text{ A}$	$\mathbf{I}_4 = -5\text{ A}$	$\mathbf{I}_6 = 5\text{ A}$

[c] The clock and television set were fed from the uninterrupted side of the circuit, that is, the $12\,\Omega$ load includes the clock and the TV set.

[d] No, the motor current drops to 5 A, well below its normal running value of 22.86 A.

[e] After fuse A opens, the current in fuse B is only 15 A.


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