ECE221 PS7 Solutions
(6.1, 6.10, 6.14, 6.20, 6.26, 6.34, 6.36, 6.39)

P 6.1 $\quad 0 \le t \le 2\,\mathrm{s}$:

$$i_L = \frac{10^3}{2.5}\int_0^t 3\times 10^{-3}e^{-4x}\,dx + 1 = 1.2\frac{e^{-4x}}{-4}\bigg|_0^t + 1$$

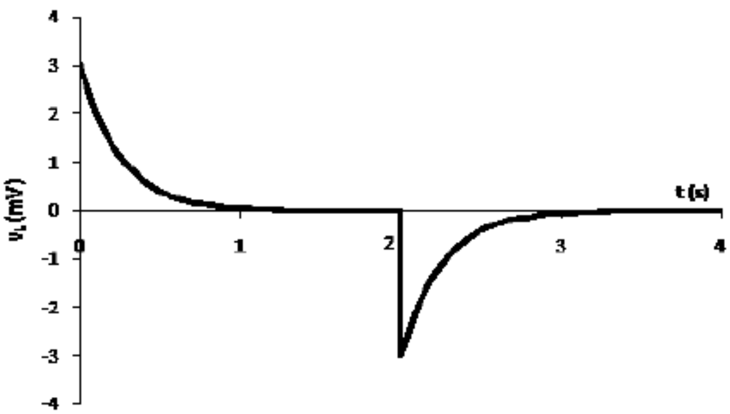
$$= -0.3e^{-4t} + 1.3\,\mathrm{A}, \qquad 0 \le t \le 2\,\mathrm{s}$$

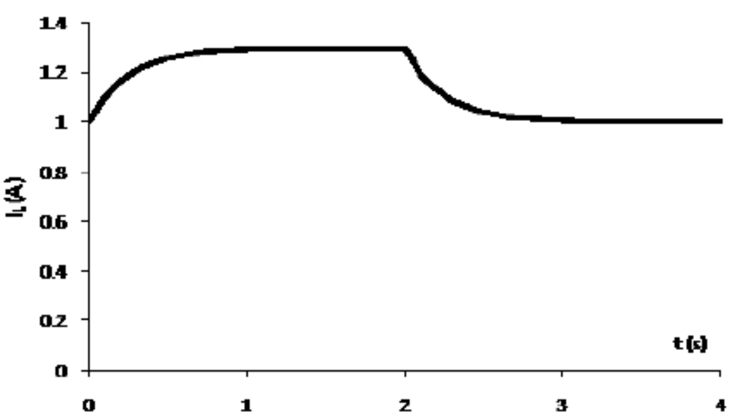
$$i_L(2) = -0.3e^{-8} + 1.3 = 1.3\,\mathrm{A}$$

$t \ge 2\,\mathrm{s}$:

$$i_L = \frac{10^3}{2.5}\int_2^t -3\times 10^{-3}e^{-4(x-2)}\,dx + 1.3 = -1.2\frac{e^{-4(x-2)}}{-4}\bigg|_2^t + 1.3$$

$$= 0.3e^{-4(t-2)} + 1\,\mathrm{A}, \qquad t \ge 2\,\mathrm{s}$$





P 6.10 $i = (B_1 \cos 4t + B_2 \sin 4t)e^{-t/2}$

$$i(0) = B_1 = 10\,\text{A}$$

$$\frac{di}{dt} = (B_1 \cos 4t + B_2 \sin 4t)(-0.5e^{-t/2}) + e^{-t/2}(-4B_1 \sin 4t + 4B_2 \cos 4t)$$

$$= [(4B_2 - 0.5B_1)\cos 4t - (4B_1 + 0.5B_2)\sin 4t]e^{-t/2}$$

$$v = 4\frac{di}{dt} = [(16B_2 - 2B_1)\cos 4t - (16B_1 + 2B_2)\sin 4t]e^{-t/2}$$

$$v(0) = 60 = 16B_2 - 2B_1 = 16B_2 - 20 \quad \therefore \quad B_2 = 5\,\text{A}$$

Thus,

$$i = (10\cos 4t + 5\sin 4t)e^{-t/2}, \text{A}, \qquad t \geq 0$$

$$v = (60\cos 4t - 170\sin 4t)e^{-t/2}\,\text{V}, \qquad t \geq 0$$

$$i(1) = -26, \text{A}; \qquad v(1) = 54.25\,\text{V}$$

$$p(1) = (-26)(54.25) = -339.57\,\text{W delivering}$$

P 6.14 [a] $i = \frac{400 \times 10^{-3}}{5 \times 10^{-6}}t = 80 \times 10^3 t \qquad 0 \le t \le 5\,\mu\text{s}$

$i = 400 \times 10^{-3} \qquad 5 \le t \le 20\,\mu\text{s}$

$i = \frac{300 \times 10^{-3}}{30 \times 10^{-6}}t - 0.5 = 10^4 t - 0.5 \qquad 20\,\mu\text{s} \le t \le 50\,\mu\text{s}$

$$\begin{aligned} q &= \int_0^{5\times10^{-6}} 8 \times 10^4 t\,dt + \int_{5\times10^{-6}}^{15\times10^{-6}} 0.4\,dt \\ &= 8 \times 10^4 \frac{t^2}{2}\Big|_0^{5\times10^{-6}} + 0.4(10 \times 10^{-6}) \\ &= 4 \times 10^4 (25 \times 10^{-12}) + 4 \times 10^{-6} \\ &= 5\,\mu\text{C} \end{aligned}$$

[b] $v = 4 \times 10^6 \int_0^{5\times10^{-6}} 8 \times 10^4 x\,dx + 4 \times 10^6 \int_{5\times10^{-6}}^{20\times10^{-6}} 0.4\,dx$

$+ 4 \times 10^6 \int_{20\times10^{-6}}^{30\times10^{-6}} (10^4 x - 0.5)\,dx$

$= 4 \times 10^6 \left[8 \times 10^4 \frac{x^2}{2}\Big|_0^{5\times10^{-6}} + 0.4x\Big|_{5\times10^{-6}}^{20\times10^{-6}} + 10^4 \frac{x^2}{2}\Big|_{20\times10^{-6}}^{30\times10^{-6}} - 0.5x\Big|_{20\times10^{-6}}^{30\times10^{-6}} \right.$

$= 4 \times 10^6 [4 \times 10^4 (25 \times 10^{-12}) + 0.4(15 \times 10^{-6})$

$+ 5000(900 \times 10^{-12} 400 \times 10^{-12}) - 0.5(10 \times 10^{-6})]$

$= 18\,\text{V}$

$v(30\,\mu\text{s}) = 18\,\text{V}$

[c] $v(50\,\mu\text{s}) = 4 \times 10^6 [10^{-6} + 6 \times 10^{-6} + 5000(2500 \times 10^{-12} - 400 \times 10^{-12})$

$- 0.5(30 \times 10^{-6})]$

$= 10\,\text{V}$

$w = \frac{1}{2}Cv^2 = \frac{1}{2}(0.25 \times 10^{-6})(10)^2 = 12.5\,\mu\text{J}$

P 6.20 $30\|20 = 12\,\text{H}$

$80\|(8+12) = 16\,\text{H}$

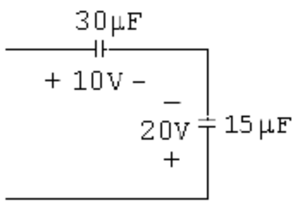
$60\|(14+16) = 20\,\text{H}$

$15\|(20+10) = 20\,\text{H}$

$L_{\text{ab}} = 5 + 10 = 15\,\text{H}$

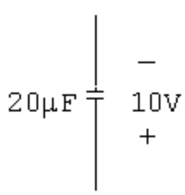
P 6.26 $\frac{1}{C_1} = \frac{1}{48} + \frac{1}{16} = \frac{1}{12}; \qquad C_1 = 12\,\mu\text{F}$

$C_2 = 3 + 12 = 15\,\mu\text{F}$

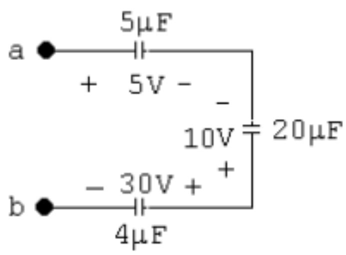


$\frac{1}{C_3} = \frac{1}{30} + \frac{1}{15} = \frac{1}{10}; \qquad C_3 = 10\,\mu\text{F}$

$C_4 = 10 + 10 = 20\,\mu\text{F}$



$\frac{1}{C_5} = \frac{1}{5} + \frac{1}{20} + \frac{1}{4} = \frac{1}{2}; \qquad C_5 = 2\,\mu\text{F}$



Equivalent capacitance is $2\,\mu\text{F}$ with an initial voltage drop of +25 V.

P 6.34
$$\frac{di_o}{dt} = (5)\{e^{-2000t}[-8000\sin 4000t + 4000\cos 4000t]$$
$$+(-2000e^{-2000t})[2\cos 4000t + \sin 4000t]\}$$
$$= e^{-2000t}\{-50{,}000\sin 4000t\}\ \text{V}$$

$$\frac{di_o}{dt}(0^+) = (1)[\sin(0)] = 0$$

$$\therefore \qquad 10\times 10^{-3}\frac{di_o}{dt}(0^+) = 0 \qquad \text{so} \qquad v_2(0^+) = 0$$

$$v_1(0^+) = 40i_o(0^+) + v_2(0^+) = 40(10) = 0 = 400\ \text{V}$$

P 6.36 [a] Rearrange by organizing the equations by di_1/dt, i_1, di_2/dt, i_2 and transfer the i_g terms to the right hand side of the equations. We get

$$4\frac{di_1}{dt} + 25i_1 - 8\frac{di_2}{dt} - 20i_2 = 5i_g - 8\frac{di_g}{dt}$$

$$-8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 80i_2 = 16\frac{di_g}{dt}$$

[b] From the given solutions we have

$$\frac{di_1}{dt} = -320e^{-5t} + 272e^{-4t}$$

$$\frac{di_2}{dt} = 260e^{-5t} - 204e^{-4t}$$

Thus,

$$4\frac{di_1}{dt} = -1280e^{-5t} + 1088e^{-4t}$$

$$25i_1 = 100 + 1600e^{-5t} - 1700e^{-4t}$$

$$8\frac{di_2}{dt} = 2080e^{-5t} - 1632e^{-4t}$$

$$20i_2 = 20 - 1040e^{-5t} + 1020e^{-4t}$$

$$5i_g = 80 - 80e^{-5t}$$

$$8\frac{di_g}{dt} = 640e^{-5t}$$

Thus,

$$-1280e^{-5t} + 1088e^{-4t} + 100 + 1600e^{-5t} - 1700e^{-4t} - 2080e^{-5t}$$

$$+1632e^{-4t} - 20 + 1040e^{-5t} - 1020e^{-4t} \stackrel{?}{=} 80 - 80e^{-5t} - 640e^{-5t}$$

$$80 + (1088 - 1700 + 1632 - 1020)e^{-4t}$$

$$+(1600 - 1280 - 2080 + 1040)e^{-5t} \stackrel{?}{=} 80 - 720e^{-5t}$$

$$80 + (2720 - 2720)e^{-4t} + (2640 - 3360)e^{-5t} = 80 - 720e^{-5t} \quad \text{(OK)}$$

$$8\frac{di_1}{dt} = -2560e^{-5t} + 2176e^{-4t}$$

$$20i_1 = 80 + 1280e^{-5t} - 1360e^{-4t}$$

$$16\frac{di_2}{dt} = 4160e^{-5t} - 3264e^{-4t}$$

$$80i_2 = 80 - 4160e^{-5t} + 4080e^{-4t}$$

$$16\frac{di_g}{dt} = 1280e^{-5t}$$

$$2560e^{-5t} - 2176e^{-4t} - 80 - 1280e^{-5t} + 1360e^{-4t} + 4160e^{-5t} - 3264e^{-4t}$$

$$+80 - 4160e^{-5t} + 4080e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$(-80 + 80) + (2560 - 1280 + 4160 - 4160)e^{-5t}$$

$$+(1360 - 2176 - 3264 + 4080)e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$0 + 1280e^{-5t} + 0e^{-4t} = 1280e^{-5t} \quad \text{(OK)}$$

P 6.39 [a] $-2\frac{di_g}{dt} + 16\frac{di_2}{dt} + 32i_2 = 0$

$$16\frac{di_2}{dt} + 32i_2 = 2\frac{di_g}{dt}$$

[b] $i_2 = e^{-t} - e^{-2t}\,\text{A}$

$$\frac{di_2}{dt} = -e^{-t} + 2e^{-2t}\,\text{A/s}$$

$$i_g = 8 - 8e^{-t}\,\text{A}$$

$$\frac{di_g}{dt} = 8e^{-t}\,\text{A/s}$$

$$\therefore \quad -16e^{-t} + 32e^{-2t} + 32e^{-t} - 32e^{-2t} = 16e^{-t}$$

[c]

$$\begin{aligned} v_1 &= 4\frac{di_g}{dt} - 2\frac{di_2}{dt} \\ &= 4(8e^{-t}) - 2(-e^{-t} + 2e^{-2t}) \\ &= 34e^{-t} - 4e^{-2t}\,\text{V}, \qquad t > 0 \end{aligned}$$

[d] $v_1(0) = 34 - 4 = 30\,\text{V};$ Also

$$\begin{aligned} v_1(0) &= 4\frac{di_g}{dt}(0) - 2\frac{di_2}{dt}(0) \\ &= 4(8) - 2(-1 + 2) = 32 - 2 = 30\,\text{V} \end{aligned}$$

Yes, the initial value of v_1 is consistent with known circuit behavior.

```
--
***************************************
* Prof. Christopher Rose              *
* Rutgers Electrical & Computer Eng.  *
* WINLAB                              *
* Technology Center of New Jersey     *
* 671 Route 1 South                   *
* North Brunswick, NJ  08902-3390     *
* voice: 732-932-6857 x643            *
* 732-932-6882  (Fax)                 *
* http://www.winlab.rutgers.edu/~crose *
***************************************
```