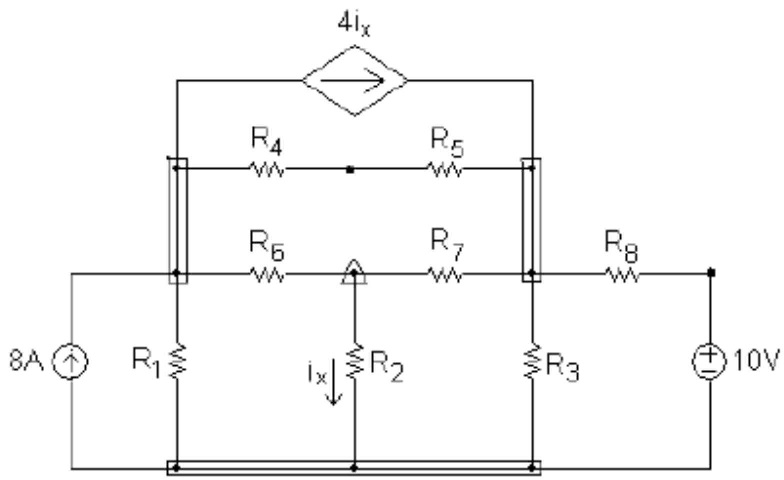
ECE221 Fall'12 PS 4 Solution

P 4.1



[a] 11 branches, 8 branches with resistors, 2 branches with independent sources, 1 branch with a dependent source

[b] The current is unknown in every branch except the one containing the 8 A current source, so the current is unknown in 10 branches.

[c] 9 essential branches – $R_4 - R_5$ forms an essential branch as does $R_8 - 10$ V. The remaining seven branches are essential branches that contain a single element.

[d] The current is known only in the essential branch containing the current source, and is unknown in the remaining 8 essential branches

[e] From the figure there are 6 nodes – three identified by rectangular boxes, two identified with single black dots, and one identified by a triangle.

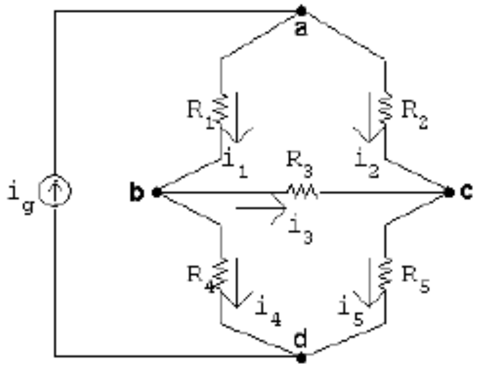
[f] There are 4 essential nodes, three identified with rectangular boxes and one identified with a triangle

[g] A mesh is like a window pane, and as can be seen from the figure there are 6 window panes or meshes.

P 4.4 [a] There are six circuit components, five resistors and the current source. Since the current is known only in the current source, it is unknown in the five resistors. Therefore there are five unknown currents.

[b] There are four essential nodes in this circuit, identified by the dark black dots in Fig. P4.4. At three of these nodes you can write KCL equations that will be independent of one another. A KCL equation at the fourth node would be dependent on the first three. Therefore there are three independent KCL equations.

[c]



Sum the currents at any three of the four essential nodes a, b, c, and d. Using nodes a, b, and c we get

$$-i_g + i_1 + i_2 = 0$$

$$-i_1 + i_4 + i_3 = 0$$

$$i_5 - i_2 - i_3 = 0$$

[d] There are three meshes in this circuit: one on the left with the components i_g, R_1, and R_4; one on the top right with components R_1, R_2, and R_3; and one on the bottom right with components R_3, R_4, and R_5. We cannot write a KVL equation for the left mesh because we don't know the voltage drop across the current source. Therefore, we can write KVL equations for the two meshes on the right, giving a total of **two** independent KVL equations.

[e] Sum the voltages around two independent closed paths, avoiding a path that contains the independent current source since the voltage across the current source is not known. Using the upper and lower meshes formed by the five resistors gives

$$R_1 i_1 + R_3 i_3 - R_2 i_2 = 0$$

$$R_3 i_3 + R_5 i_5 - R_4 i_4 = 0$$

P 4.8 $-6 + \frac{v_1}{40} + \frac{v_1 - v_2}{8} = 0$

$$\frac{v_2 - v_1}{8} + \frac{v_2}{80} + \frac{v_2}{120} + 1 = 0$$

Solving, $v_1 = 120$ V; $v_2 = 96$ V

CHECK:

$$p_{40\Omega} = \frac{(120)^2}{40} = 360 \text{ W}$$

$$p_{8\Omega} = \frac{(120-96)^2}{8} = 72 \text{ W}$$

$$p_{80\Omega} = \frac{(96)^2}{80} = 115.2 \text{ W}$$

$$p_{120\Omega} = \frac{(96)^2}{120} = 76.8 \text{ W}$$

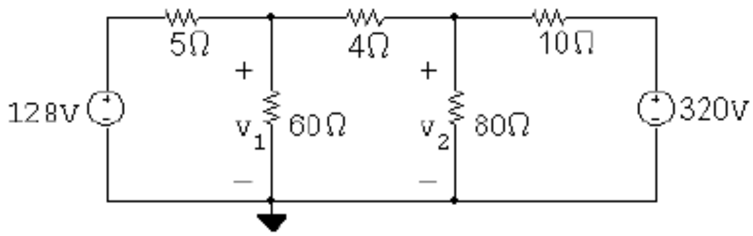
$$p_{6\text{A}} = -(6)(120) = -720 \text{ W}$$

$$p_{1\text{A}} = (1)(96) = 96 \text{ W}$$

$$\sum p_{\text{abs}} = 360 + 72 + 115.2 + 76.8 + 96 = 720 \text{ W}$$

$$\sum p_{\text{dev}} = 720 \text{ W} \quad \text{(CHECKS)}$$

P 4.13 [a]



$$\frac{v_1 - 128}{5} + \frac{v_1}{60} + \frac{v_1 - v_2}{4} = 0$$

$$\frac{v_2 - v_1}{4} + \frac{v_2}{80} + \frac{v_2 - 320}{10} = 0$$

In standard form,

$$v_1\left(\frac{1}{5} + \frac{1}{60} + \frac{1}{4}\right) + v_2\left(-\frac{1}{4}\right) = \frac{128}{5}$$

$$v_1\left(-\frac{1}{4}\right) + v_2\left(\frac{1}{4} + \frac{1}{80} + \frac{1}{10}\right) = \frac{320}{10}$$

Solving, $v_1 = 162$ V; $\quad v_2 = 200$ V

$$i_a = \frac{128 - 162}{5} = -6.8 \text{ A}$$

$$i_b = \frac{162}{60} = 2.7 \text{ A}$$

$$i_c = \frac{162 - 200}{4} = -9.5 \text{ A}$$

$$i_d = \frac{200}{80} = 2.5 \text{ A}$$

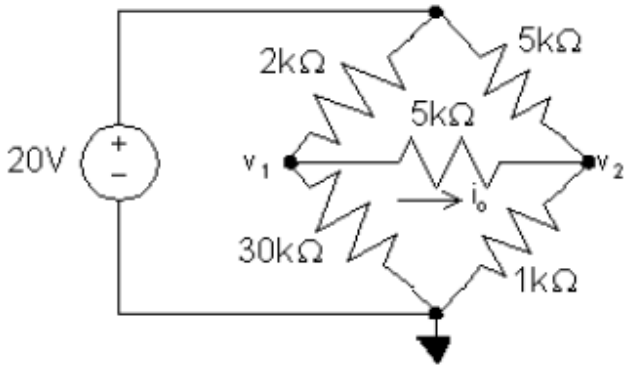
$$i_e = \frac{200 - 320}{10} = -12 \text{ A}$$

[b] $p_{128V} = -(128)(-6.8) = 870.4$ W (abs)

$p_{320V} = (320)(-12) = -3840$ W (dev)

Therefore, the total power developed is 3840 W.

P 4.21



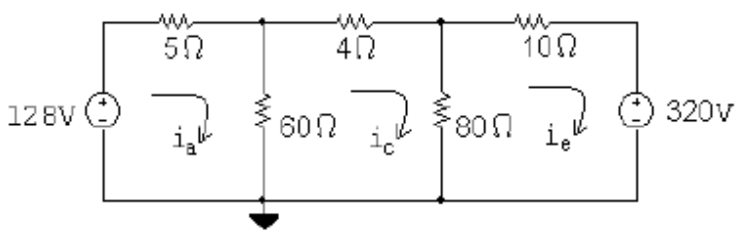
$$\frac{v_1}{30{,}000} + \frac{v_1 - v_2}{5000} + \frac{v_1 - 20}{2000} = 0 \qquad \text{so} \qquad 22v_1 - 6v_2 = 300$$

$$\frac{v_2}{1000} + \frac{v_2 - v_1}{5000} + \frac{v_2 - 20}{5000} = 0 \qquad \text{so} \qquad -v_1 + 7v_2 = 20$$

Solving, $v_1 = 15$ V; $\qquad v_2 = 5$ V

Thus, $i_o = \dfrac{v_1 - v_2}{5000} = 2$ mA

P 4.32 [a]



The three mesh current equations are:

$$-128 + 5i_\text{a} + 60(i_\text{a} - i_\text{c}) = 0$$

$$4i_\text{c} + 80(i_\text{c} - i_\text{e}) + 60(i_\text{c} - i_\text{a}) = 0$$

$$320 + 80(i_\text{e} - i_\text{c}) + 10i_\text{e} = 0$$

Place these equations in standard form:

$$i_\text{a}(5 + 60) + i_\text{c}(-60) + i_\text{e}(0) = 128$$

$$i_\text{a}(-60) + i_\text{c}(4 + 80 + 60) + i_\text{e}(-80) = 0$$

$$i_\text{a}(0) + i_\text{c}(-80) + i_\text{e}(80 + 10) = -320$$

Solving, $i_\text{a} = -6.8$ A; $i_\text{c} = -9.5$ A; $i_\text{e} = -12$ A
Now calculate the remaining branch currents:

$$i_\text{b} = i_\text{a} - i_\text{c} = 2.7 \text{ A}$$

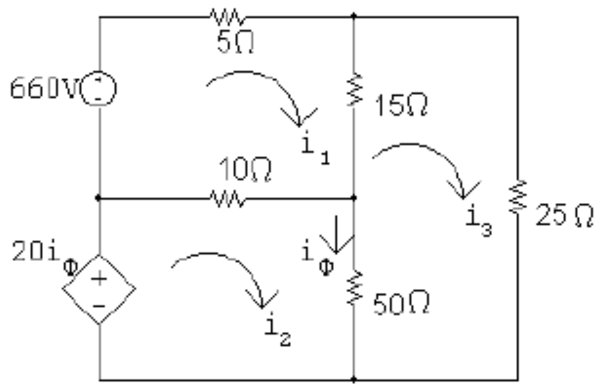
$$i_\text{d} = i_\text{c} - i_\text{e} = 2.5 \text{ A}$$

[b] $p_{128\text{V}} = -(128)i_a = -(128)(-6.8) = 870.4$ W (abs)

$p_{320\text{V}} = (320)i_e = (320)(-12) = -3840$ W (dev)

Thus, the power developed in the circuit is 3840 W. Note that the resistors cannot develop power!

P 4.39



$$660 = 30i_1 - 10i_2 - 15i_3$$

$$20i_\phi = -10i_1 + 60i_2 - 50i_3$$

$$0 = -15i_1 - 50i_2 + 90i_3$$

$$i_\phi = i_2 - i_3$$

Solving, $i_1 = 42$ A; $\quad i_2 = 27$ A; $\quad i_3 = 22$ A; $\quad i_\phi = 5$ A

$$20i_\phi = 100 \text{ V}$$

$$p_{20i_\phi} = -100i_2 = -100(27) = -2700 \text{ W}$$

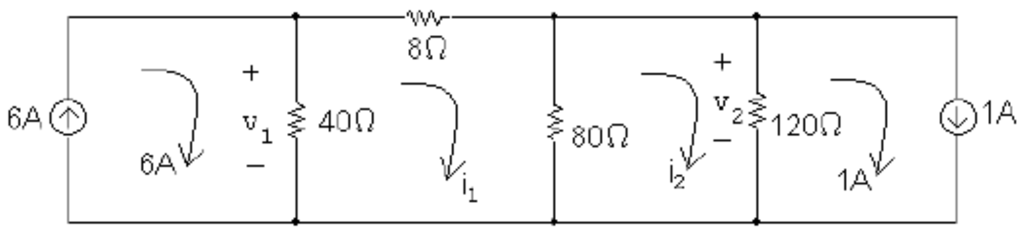
$$\therefore \quad p_{20i_\phi} \text{ (developed) } = 2700 \text{ W}$$

CHECK:

$$p_{660\text{V}} = -660(42) = -27{,}720 \text{ W (dev)}$$

$$\begin{aligned} \therefore \quad \sum P_{\text{dev}} &= 27{,}720 + 2700 = 30{,}420 \text{ W} \\ \sum P_{\text{dis}} &= (42)^2(5) + (22)^2(25) + (20)^2(15) + (5)^2(50) + \\ &\quad (15)^2(10) \\ &= 30{,}420 \text{ W} \end{aligned}$$

P 4.41



Mesh equations:

$$128i_1 - 80i_2 = 240$$

$$-80i_1 + 200i_2 = 120$$

Solving,

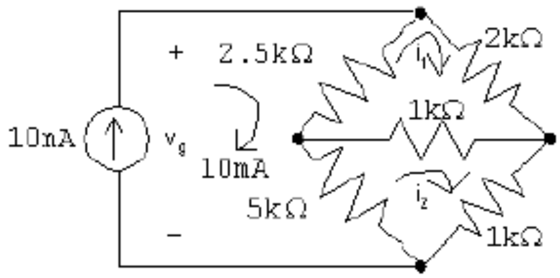
$$i_1 = 3 \text{ A}; \qquad i_2 = 1.8 \text{ A}$$

Therefore,

$$v_1 = 40(6 - 3) = 120 \text{ V}; \qquad v_2 = 120(1.8 - 1) = 96 \text{ V}$$

P 4.53 [a] There are three unknown node voltages and only two unknown mesh currents. Use the mesh current method to minimize the number of simultaneous equations.

[b]



The mesh current equations:

$$2500(i_1 - 0.01) + 2000i_1 + 1000(i_1 - i_2) = 0$$

$$5000(i_2 - 0.01) + 1000(i_2 - i_1) + 1000i_2 = 0$$

Place the equations in standard form:

$$i_1(2500 + 2000 + 1000) + i_2(-1000) = 25$$

$$i_1(-1000) + i_2(5000 + 1000 + 1000) = 50$$

Solving, $i_1 = 6$ mA; $i_2 = 8$ mA

Find the power in the 1 kΩ resistor:

$i_{1k} = i_1 - i_2 = -2$ m A

$p_{1k} = (-0.002)^2(1000) = 4$ mW

[c] No, the voltage across the 10 A current source is readily available from the mesh currents, and solving two simultaneous mesh-current equations is less work than solving three node voltage equations.

[d] $v_g = 2000i_1 + 1000i_2 = 12 + 8 = 20$ V

$p_{10mA} = -(20)(0.01) = -200$ m W

Thus the 10 mA source develops 200 mW.

```
--
****************************************
* Prof. Christopher Rose               *
* Rutgers Electrical & Computer Eng.   *
* WINLAB                               *
* Technology Center of New Jersey      *
* 671 Route 1 South                    *
* North Brunswick, NJ  08902-3390      *
* voice: 732-932-6857 x643             *
* 732-932-6882  (Fax)                  *
* http://www.winlab.rutgers.edu/~crose *
****************************************
```