School of Engineering
Department of Electrical and Computer Engineering

332:221 **Principles of Electrical Engineering I** **Fall 2012**

Quiz 3 (THE END!)

No calculators, no books, no class notes, no nuttin'! Just a pencil/pen, your THREE SIDES of 8.5×11 ***cheat sheet and you. Final answers must appear in the appropriate box. Show your work outside the box.***

1. *(10 pts)* **Basic Circuits:**

   All parts refer to FIGURE 1.

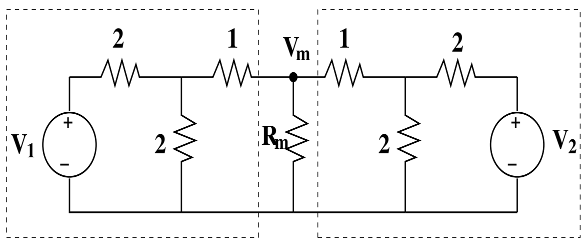


Figure 1: Figure for problem 1

(a) *(2 pts)* What is the *minimum number* of nodes (other than V_m and ground) necessary to uniquely determine all currents and voltages in the circuit using the node voltage method. 2

(b) *(2 pts)* What is *minimum number* of current meshes in the circuit that will allow you to determine all currents and voltages in the circuit. 4

(c) *(2 pts)* If $R_m = 2$, what is the impedance seen by source V_1? 3

(d) *(2 pts)* If $R_m = 2$, what is the impedance seen by source V_2? 3

(e) *(2 pts)* Assume $V_1 = V_2 = V$. What is V_m for arbitrary R_m? $\frac{V}{2} \frac{R_m}{R_m+1}$

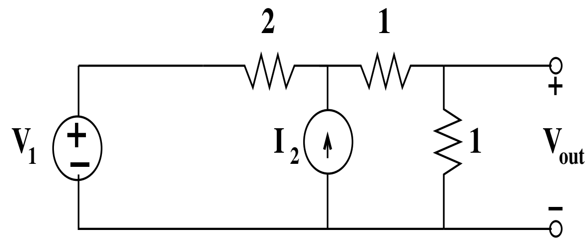


Figure 2: Figure for problem 2

2. *(10 pts)* **Basic Thevenin/Norton:**

   All questions refer to FIGURE 2.

   (a) *(5 pts)* Given $V_1 = 8$ and $I_2 = 2$, please provide a thevenin equivalent for the circuit shown.

   $$V_{th} = \frac{V_1}{4} + \frac{I_2}{2} = 3$$
   $$R_{th} = 0.75$$

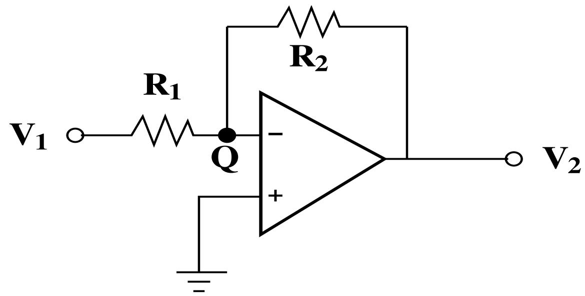
   (b) *(5 pts)* Given $V_1 = 9$ and $I_2 = 3$, please provide a norton equivalent for the circuit shown.

   $$I_{th} = \frac{V_1}{3} + \frac{2I_2}{3} = 5$$

   $$R_{th} = 0.75$$

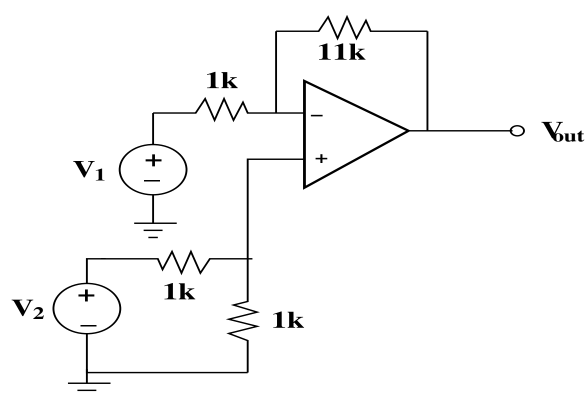
3. *(10 pts)* **Basic Op-Amps:**

(a) *(5 pts)* Please provide a labeled diagram for an amplifier with gain $G = -10$ and input impedance $30k\Omega$ when the op-amp operates in its linear range. You may use only a single op-amp. For full credit, you must explain how your circuit meets the input impedance specification.



with $R_1 = 30k$ ***and*** $R_2 = 300k$***. Because*** Q ***is a virtual ground, the input sees a*** $30k$ ***input impedance.***

(b) *(5 pts)* Please provide a labeled diagram for a circuit which produces $V_{\text{out}} = 6V_2 - 11V_1$. You may use only a single op-amp. If your circuit is operating linearly, what impedance is seen by input V_2? (Explain your impedance answer.)



The input impedance from V_2 ***is*** $2k$ ***since the op-amp does not load the voltage divider.***

4. *(10 pts)* **Basic RLC/Impedance/Power:**

All parts refer to FIGURE 3.

(a) *(5 pts)*

Element A is an inductor with value L. Element B is a resistor with value R. Element C is a capacitor with value C_1. Element D is a capacitor with with value C_2. What is the impedance of this circuit as seen from the terminals?

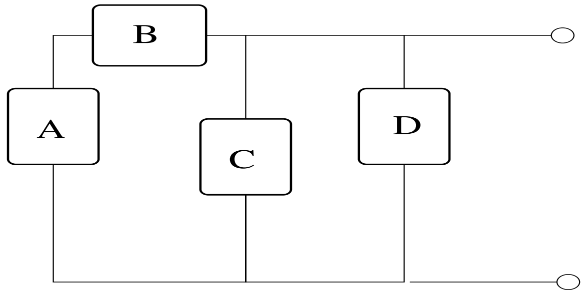


Figure 3: Figure for problem 4

$$Z_{eq} = \frac{Lj\omega + R}{L(C_1+C_2)(j\omega)^2 + R(C_1+C_2)j\omega + 1}$$

(b) *(5 pts)*
If phasor current $Ie^{j\phi}$ is applied at the terminals, what is the average power dissipated in the resistor?

Current divider yields

$$\mathbf{I}_R = Ie^{j\phi}\frac{1}{L(C_1+C_2)(j\omega)^2 + R(C_1+C_2)j\omega + 1}$$

so that

$$\bar{P}_R = \frac{1}{2}I^2R\frac{1}{(1 - L(C_1+C_2)\omega^2)^2 + (R(C_1+C_2)\omega)^2}$$

5. *(10 pts)* **Basic Phasors and Transfer Functions**

(a) *(1 pts)* Y/N: $e^{j\pi} = -1$: **Y**

(b) *(1 pts)* Y/N: $\frac{1}{\sqrt{2}}(e^{j\frac{\pi}{2}} - 1) = e^{-j\frac{\pi}{2}}$: **N**

(c) *(1 pts)* Y/N: $\cos(\omega t + \phi) = e^{j\omega t + j\phi}$: **N**

(d) *(3 pts)*

$$H(j\omega) = \frac{dj\omega}{a(j\omega)^2 + bj\omega + c}$$

where $a, b, c\&d$ are real. If $\omega = \sqrt{\frac{c}{a}}$ what is $\Re\{H(j\omega)\}$? $\frac{d}{b}$

(e) *(2 pts)* If at some frequency ω, $H(j\omega) = 1 + j$ and the input to the system is $\sin \omega t$, what is the output in time domain?

$$\sqrt{2}\sin(\omega t + \pi/4) = \sqrt{2}\cos(\omega t - \pi/4)$$

(f) *(2 pts)* If at some frequency ω, $H(j\omega) = \frac{1}{\sqrt{2}}(1 - j)$ and the input to the system is $\cos \omega t$, what is the output in phasor notation?

$$e^{-j\pi/4}$$

6. *(15 pts)* **Gyrations:** You have been asked to design a circuit which has a $1mH$ inductor and $1k$ resistor in series with the output of an op-amp (whose $\pm V_{cc} = \pm 10$). The circuit works just fine if you slowly turn on the supply voltage. However, if you flip the on-switch, invariably a whisp of smoke curls up from the op-amp and the circuit no longer works. You decide to measure the currents in your circuit and find that when the power is suddenly switched on, the current through the inductor goes from 0 to 1 amp within $1\mu s$ whereas when the power is ramped up slowly, the current in the inductor rises a factor of 100 more slowly.

(a) *(2 pts))*

Why is the op-amp destroyed when the power is flipped on suddenly and not when power is slowly ramped up?

Flip-on: voltage across the inductor is $V = L\frac{d}{dt}I = 10^3 V$**.**
Ramp-on: voltage across the inductor is $V = L\frac{d}{dt}I = 10V$**.**

(b) *(8 pts))*

Consider the circuit shown in FIGURE 4. Please derive the sinusoidal steady state

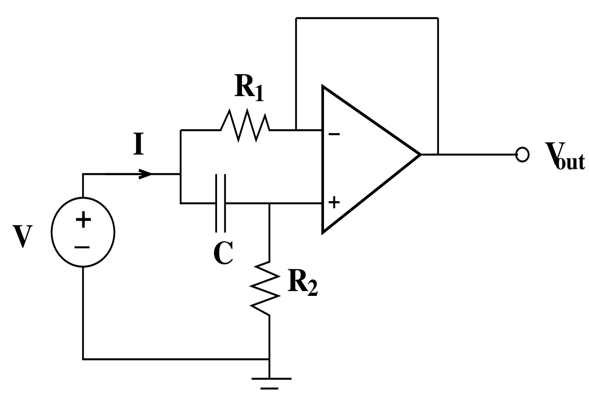


Figure 4: Gyrator circuit

impedance as seen from input V for this circuit. Owing to negative feedback, you may assume the circuit is stable and operating in its linear range.

We assume that negative feedback assures $V_p = V_n$. We then have $V_p = V\frac{R_2Cj\omega}{R_2Cj\omega+1} = V_{out}$. This means that

$$I = \frac{V}{\frac{1}{Cj\omega} + R_2} + \frac{1}{R_1}V\left(1 - \frac{R_2Cj\omega}{R_2Cj\omega+1}\right) = V\frac{Cj\omega + \frac{1}{R_1}}{R_2Cj\omega+1}$$

From which we deduce

$$Z_{in} = \frac{R_1R_2Cj\omega + R_l}{R_1Cj\omega + 1}$$

(c) *(5 pts))*

If we assume that we choose component values so that we always have $R_1Cj\omega \ll 1$, explain how the circuit can be used to save your filter circuit op-amp from power-on transients?

This circuit (called a "gyrator") behaves like a series combination of a resistor R_1 and an inductor $L = R_1R_2C$. However, since the op-amp which comprises the gyrator can never generate a voltage larger than $\pm V_{cc}$, we're safe from ugly kilovolt transients.

7. *(15 pts)* **Emma and Castor Join Forces:** You know that Emma the Electrical Engineer and her arch nemesis, the wily Dr. Castor Canadensis don't like each other at all. But rather than drag you through their plans for each other's destruction they have called a truce – in order torture you more directly.

(a) *(5 pts)*

Castor: the following lovely circuit diagram is modeled after the pattern on my enormous and powerful beaver tail! Impressive, no?! When I apply phasor voltage $\mathbf{V} = 1$

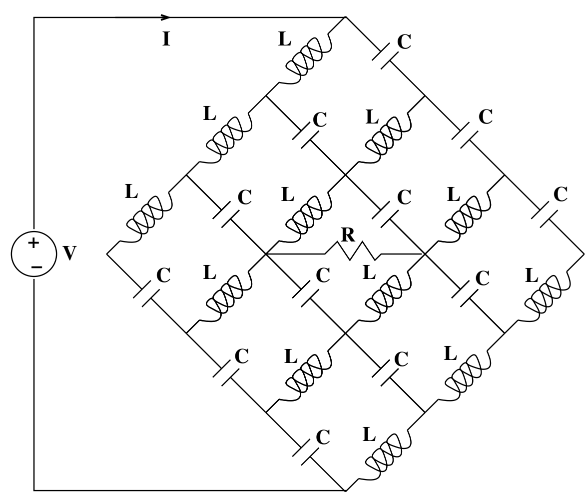


Figure 5: Beaver Tail Circuit

at frequency ω_0, I measure an input current waveform $i(t) = 10 \sin \omega_0 t$ corresponding to the phasor current **I** as shown. What is the voltage across the resistor, $v_R(t)$? Why?

Current and voltage are $\pi/2$ out of phase so no average power flows into the circuit. If any voltage appeared across the resistor, some power would be dissipated, so the voltage across the resistor must be identically zero.

(b) *(5 pts)*

Emma: So, you let Castor show you his tail, huh? Do you now understand why I hate that beaver? I hate him so much I removed all his capacitors and replaced them with diodes as shown in FIGURE 6. If V is a positive DC voltage what is the voltage across

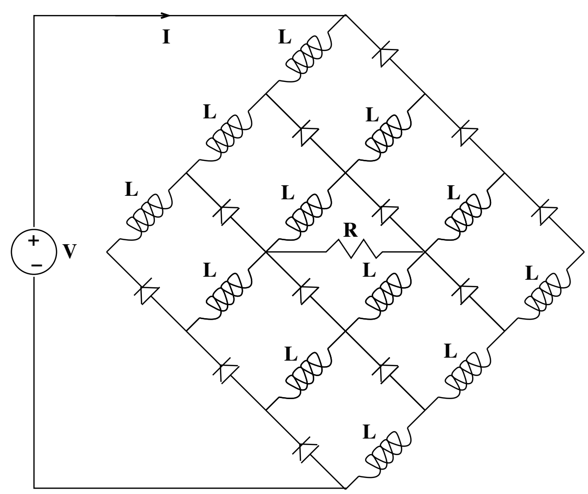


Figure 6: Beaver Tail Diode

the resistor? Why? If V is negative, does your answer change? Why/why not?

The inductors become shorts at DC and if $V > 0$ then the diodes are open – which means no current gets to the resistor. If $V < 0$ then the diodes are shorts, but then the resistor is shorted too. So at DC the voltage across the resistor is zero.

(c) *(5 pts)*

Suppose all the diodes in the previous circuit are replaced by shorts, but the circuit no longer operates only at DC. Draw an equivalent circuit and derive the transfer function from the voltage input to the voltage across the resistor.

The three tiers of inductors are all in parallel and can be replaced by an equivalent $L/4$. However, the R is in parallel with the middle tier, so the resultant circuit from the perspective looks like two inductors of inductance $L/4$ in series with a parallel combination of $L/4$ and R. So the impedance of the circuit is

$$Z_{eq} = \frac{Lj\omega}{2} + \frac{\frac{RLj\omega}{4}}{R + \frac{Lj\omega}{4}} = \frac{\frac{Lj\omega}{2}(R + \frac{Lj\omega}{4}) + \frac{RLj\omega}{4}}{R + \frac{Lj\omega}{4}}$$

The phasor current is then

$$\mathbf{I} = \mathbf{V}/Z_{eq}$$

and the current through the resistor is

$$\mathbf{I}_R = \mathbf{V}/Z_{eq}\frac{\frac{Lj\omega}{4}}{R + \frac{Lj\omega}{4}} = \mathbf{V}\frac{\frac{Lj\omega}{4}}{\frac{Lj\omega}{2}(R + \frac{Lj\omega}{4}) + \frac{RLj\omega}{4}}$$

so the voltage across the resistor is

$$\mathbf{V}_R = \mathbf{V}\frac{R}{2(R + \frac{Lj\omega}{4}) + R}$$

8. *(20 pts)*

EE's Rule ALL Engineering!:

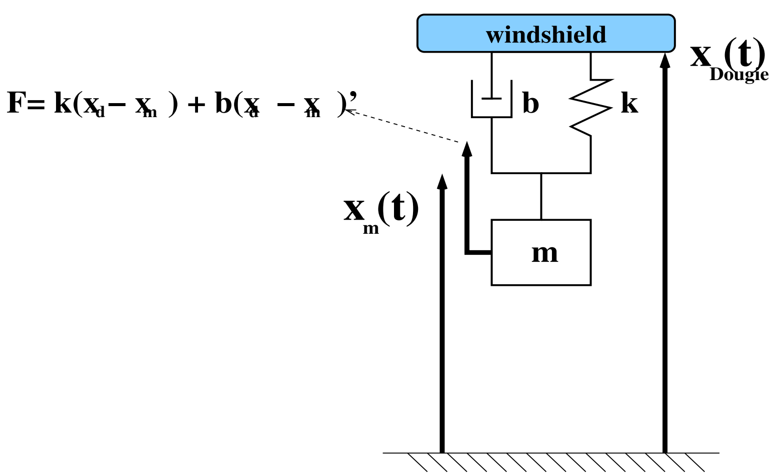


Figure 7: Figure for problem 8

I have an hour-long commute from NY into Rutgers (though on bad days – which occur too often – it can be 2 hours). Almost needless to say, I listen to a LOT of music – it has a calming effect and keeps me from committing vehiclecide. :)

On a recent trip, I was listening to *Teach Me How to Dougie* by Cali Swag District (the "clean" version, of course! :)) at high volume. If you've heard the song, you know it employs bass drums at different pitches.

[BOOM!]
They be like Smoove (what?)
Can you teach me how to Dougie? **[BOOM!]**
You know why? **[BOOM!]**
Cuz all the girls love it **[BOOM!]**
...
Now you just do-you
And I'ma do me (all-day) **[BOOM!]**
...

where the [**BOOM!**] is a particularly low frequency, heart-stopping, but still pleasant drum thump. I noticed "Dougie" had an odd effect on my rear view mirror when trying to merge in between two semis to get off at Exit 9. During the [**BOOM!**] the semi behind me began to bounce violently. I thought The Dougie had driven the bridge into resonance and it was collapsing – for a millisecond of so. :) Of course, being a card-carrying gnerd, I realized it was my MIRROR that had been driven into resonance, so I decided to analyze the situation and figure out how much my mirror weighs (without snapping it off the supporting arm, of course!).

First, I drew the diagram of FIGURE 7 which shows a mass m (my mirror), a spring k (the arm which holds my mirror) and a damper/dashpot b (the lossiness of the spring – notice how when you "work" a piece of metal like a paper clip, it gets warm?). Basic physics gave me the following differential equation:

$$m\frac{d^2}{dt^2}x_m(t) + b\frac{d}{dt}\left[x_m(t) - x_{\text{Dougie}}(t)\right] + k\left[x_m(t) - x_{\text{Dougie}}(t)\right] = 0$$

where $x_m(t)$ is the position of the mass (my mirror) and $x_{\text{Dougie}}(t)$ is the sound-induced position of the windshield. I then rewrote the equation as:

$$m\ddot{x}_m + b\dot{x}_m + kx_m = b\dot{x}_{\text{Dougie}} + kx_{\text{Dougie}}$$

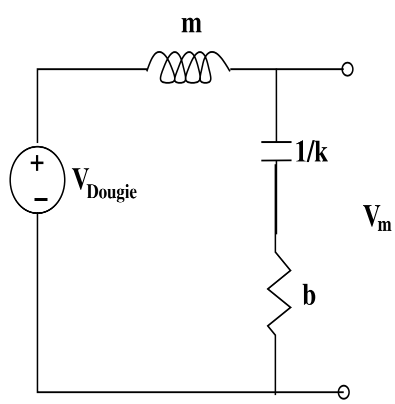
using the "dot" derivative notation.

(a) *(5 pts)* Assuming the input is always in sinusoidal steady state at frequency ω, please write down the transfer function from input $x_{\text{Dougie}}(t)$ to output $x_m(t)$.

HINT: Do not mix frequency and time domains or you will incur my UNDYING WRATH!

$$H(j\omega) = \frac{k+bj\omega}{m(j\omega)^2+bj\omega+k}$$

(b) *(10 pts)* Assume that $x_m(t)$ is analogous to the voltage across some component in a circuit and $x_{\text{Dougie}}(t)$ is analogous to a voltage source. Please draw a circuit with a voltage input $x_{\text{Dougie}}(t)$, appropriate values of resistance, inductance and capacitance and voltage across some circuit segment $x_m(t)$ which produces the same differential equation as the mass, spring, dashpot combination shown in FIGURE 7.



(c) *(5 pts)*

If the springiness of the arm which holds my mirror is $k = 720\pi^2 N/m$ and the frequency of the [**BOOM!**] is 60π radians, what is the approximate mass of my mirror?

NOTE: A correct guess gets exactly one point. Your answer needs to be consistent with your previous work.

$\sqrt{k/m} = \omega$ ***so*** $m = \frac{k}{\omega^2} = 0.2kg$