Principles of Electrical Engineering I – TOOLS

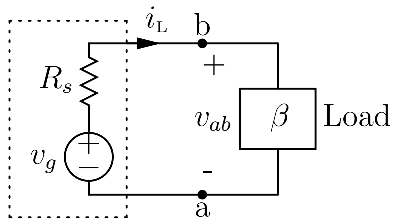
Fundamental Laws: Kirchoff's current and voltage laws
Using the above fundamental laws, we have the following techniques or tools that we can utilize to do circuit analysis:

- Voltage and current divider rules,
- Series and parallel combination of resistances,
- Δ–Y transformations,
- Node voltage method,
- Mesh current method,
- Source transformations,
- Thevenin and Norton equivalents,
- Superposition theorem.

Some of the above tools are direct analysis methods, and some others are tools to simplify network analysis. The name of the game in circuit or network simplification is constructing **network equivalents** which when appropriately done can simplify the analysis, while providing more insight into the network behavior.
Not only, we need to be very familiar with all the above tools, but also we need to know when to use what tool. Analogous to a mechanic using correctly a particular tool for a particular task, we need to learn to pick correctly an appropriate tool that suits a particular circuit. This comes only by experience. To get such an experience, one needs to do a lot of home-work problems.
Once you learn driving an automobile, you can drive on any road. Once you understand and learn fundamental methods of circuit analysis, you can solve any circuit. Two most commonly used fundamental methods of circuit analysis are **Node voltage method** and **Mesh current method**. You need to understand the intricacies of these two methods very well.

Source Transformations

Voltage Source in series with a resistance



Defining equation betwen v_{ab} and i_L:

$$\boxed{v_{ab} = v_g - i_L R_s}$$

Grtaphical representation of the defining equation:

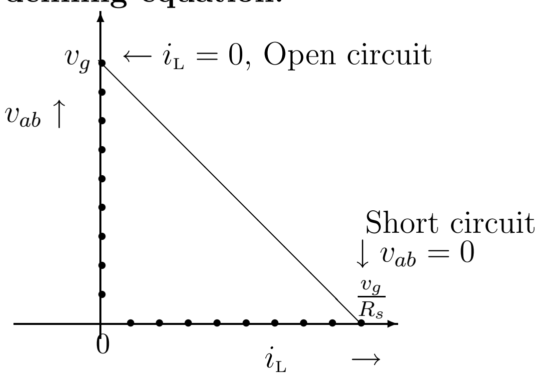
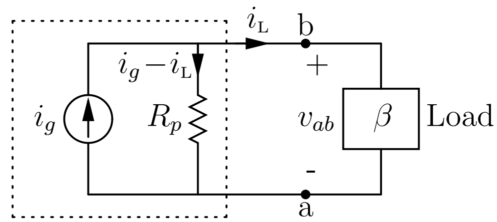


Figure 1

Current Source in parallel with a resistance



Defining equation betwen v_{ab} and i_L:

$$\boxed{v_{ab} = i_g R_p - i_L R_p}$$

Grtaphical representation of the defining equation:

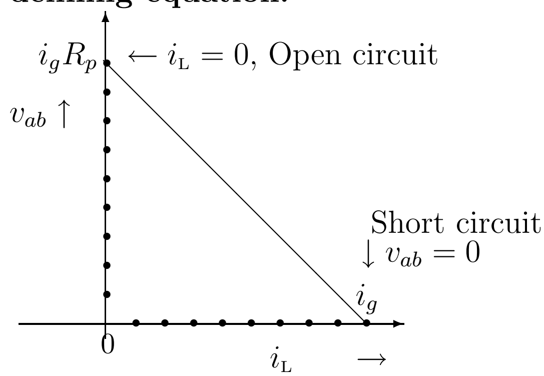


Figure 2

When can the two circuits be equivalent with respect to the terminals a and b?

They are equivalent if v_{ab} versus i_L characteristics of both are identical.

$$v_g = i_g R_p \quad \text{and} \quad \frac{v_g}{R_s} = i_g.$$

$$\frac{i_g R_p}{R_s} = i_g \;\Rightarrow\; \boxed{R_p = R_s}.$$

$$\boxed{\frac{v_g}{R_s} = i_g \;\Rightarrow\; v_g = i_g R_p}$$

Source Transformations – Example

Example. Determine the current i_0 shown in Figure (a) given below.

Think of how equivalents are determined

A series of transformations are given below.

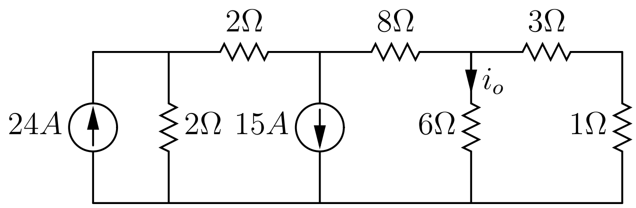


Fig. (a)

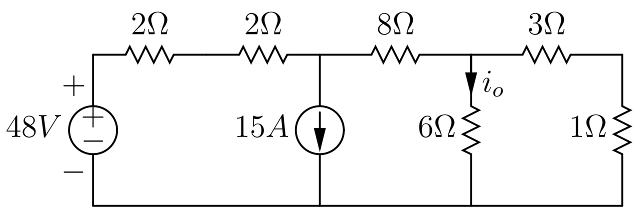


Fig. (b)

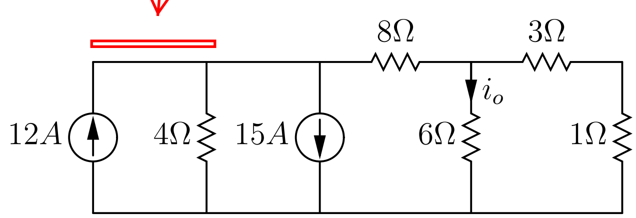


Fig. (c)

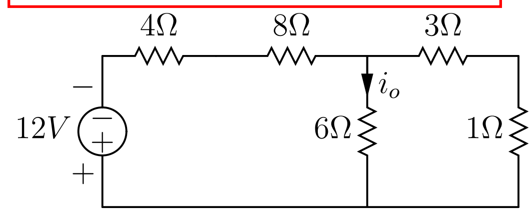


Fig. (d)

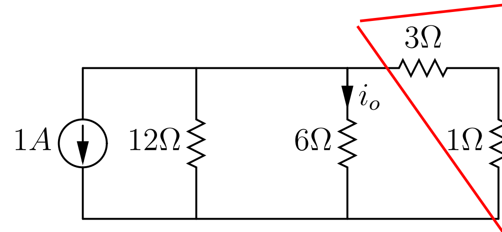


Fig. (e)

Combine into one resistance

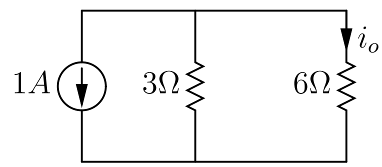


Fig. (f)

Combine the current sources into one source, and then use source transformation to get Figure)d)

Example: Our interest in this problem is to determine the currents i_x and i_y in the circuit shown in Figure 1. For this purpose we simplify the circuit as much as we can, solve the simple circuit and then come back to solve the given circuit.

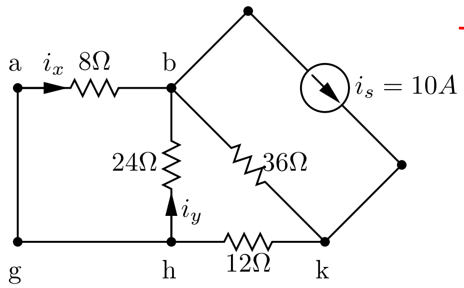


Figure 1

Source transformation

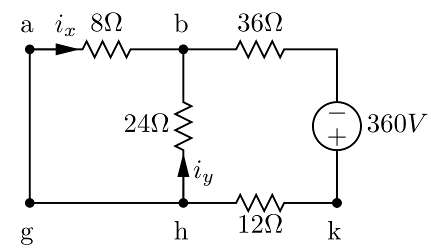


Figure 2

Source Transformation

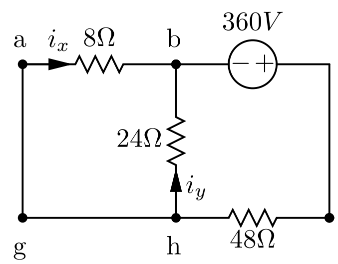


Figure 3

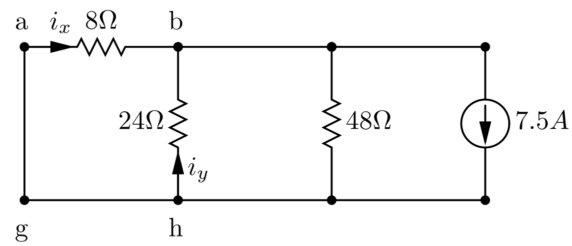


Figure 4

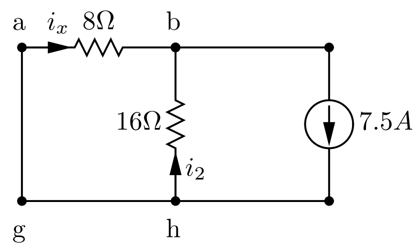


Figure 5

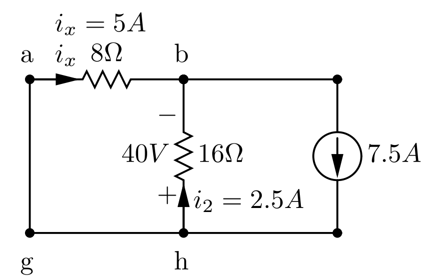


Figure 6

In the above analysis, we successively simplified the given circuit into simpler circuits. Finally, we solved a simple circuit as in Figure 6. To determine other currents in the circuit of Figure 1, we need to trace back from the simple circuit of Figure 6 to the circuit of Figure 5, and so on.

Let us consider the circuit of Figure 4 as re-drawn below on the left side. We can evaluate the current from node h to node b via 24Ω in two ways. Knowing the voltage across 24Ω as 40 V, we note the current from node h to node b as $\frac{5}{3}$ A by simple Ohm's law. Or we can evaluate it by current division rule by knowing the current 2.5 A splits into two parts, one part going through 24Ω and the other part going through 48Ω. This enables us to determine all currents in the circuit of Figure 1 as shown below on the right side.

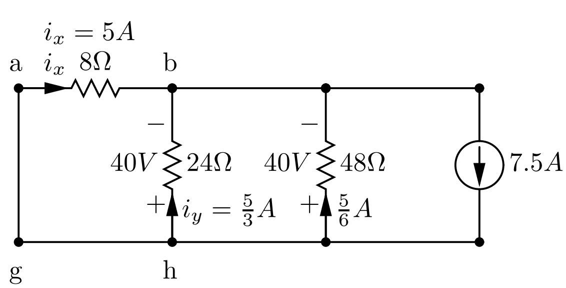


Figure 4 re-drawn

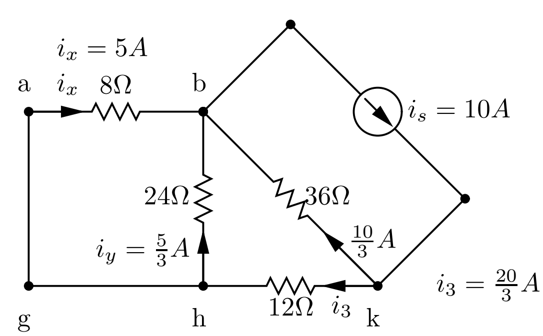


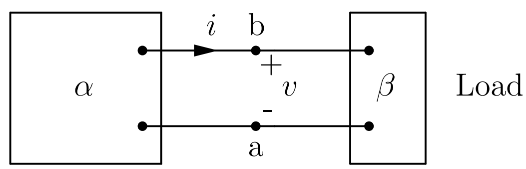
Figure 1 re-drawn

Principles of Electrical Engineering I

A Caution About Circuit Equivalents

Caution: The circuit α in Figure is a fixed circuit and the circuit β is a load on the circuit α. Suppose we intend to determine an equivalent of the fixed circuit α with respect to the terminals a and b. This can be done only when the circuit α does not have **any controlling variables** that control some dependent source which is in the circuit β. The reason is simple. If you attempt to do so, you will immediately realize that the access to the controlling variable is lost and you will not know how to characterize the dependent source which is in the circuit β.

This caution applies to Thevenin and Norton equivalents as well as Source Transformations.



Figure

Principles of Electrical Engineering I

Thevenin Theorem

Consider the interconnection of two circuits α and β as shown in Figure 1. The circuit α is a given circuit while the circuit β is a load on the circuit α. We would like to analyze the interconnected circuit to determine v and i at the terminals a and b where the two individual circuits are interconnected. We would like to do so for different types of loads (i.e. as the circuit β changes). This implies that we need to repeat the circuit analysis to determine v and i every time the circuit β changes.

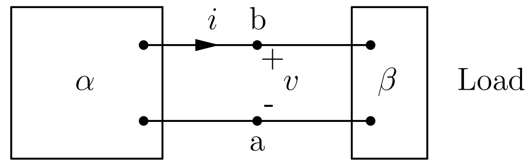
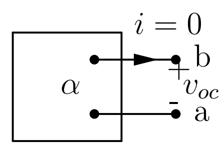


Figure 1

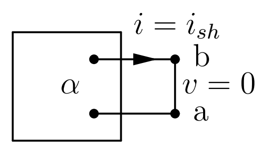
To reduce the computational burden of repetitive circuit analysis, Thevenin came up with a remarkable and highly useful result which gives a simple equivalent circuit for the circuit α. Before stating Thevenin's results, let us impose a mild condition on the circuit α.

Assumption: Let α be a <u>linear</u> circuit which could be passive or active. If it is active, let both the controlling and controlled branches be contained within it. Let there be no restrictions on the circuit β.

Open circuit voltage v_{oc}:
The voltage across the terminals a and b when the load β is an open circuit and thus draws no current can obviously be called as the open circuit voltage, and it is often denoted by v_{oc}.



Short circuit current i_{sh}:
The current from b to a when the load β is a short circuit and thus has no voltage across it can obviously be called as the short circuit current, and it is often denoted by i_{sh} or i_{sc} as in the text book.



Thevenin's Theorem: The circuit α, however complex it might be, is equivalent to a voltage source (denoted by v_{Th}) in series with a resistance (denoted by R_{Th}) between the terminals a and b as depicted in Figure 2. The voltage source v_{Th} equals the open circuit voltage v_{oc}. The resistance R_{Th} equals the ratio of open circuit voltage v_{oc} to the short circuit current i_{sh}. It is also the resistance of the circuit α seen at the terminals a and b when all the **independent** voltage as well as current sources in the circuit α are set to zero.

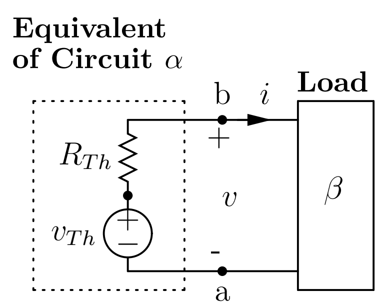


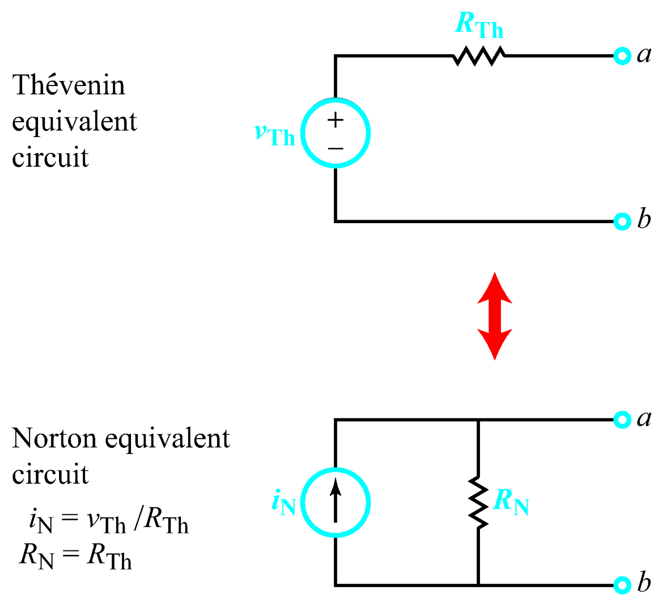
Figure 2

Proof of Thevenin's Theorem:

If we accept the fact that the circuit α, however complex it might be, can be represented by the simple circuit of Figure 2, we can then easily prove $v_{Th} = v_{oc}$ and $R_{Th} = \frac{v_{oc}}{i_{sh}}$.

Obviously, if the circuit β is an open circuit, the voltage v across the terminals a and b is v_{oc} and v_{Th} respectively in Figures 1 and 2. Then for the equivalence of the circuits in Figures 1 and 2, we must have $v_{Th} = v_{oc}$. Let us next assume that the circuit β is a short circuit. Then the current i is i_{sh} in Figure 1, and it is $\frac{v_{oc}}{R_{Th}}$ in Figure 2. This implies that for the equivalence of the circuits in Figures 1 and 2, we must have $R_{Th} = \frac{v_{oc}}{i_{sh}}$.

Thévenin-Norton Equivalency



Equivalence between Thévenin and Norton equivalent circuits.

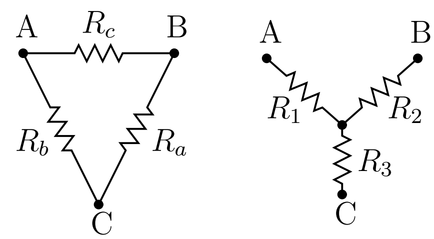
332:221 Principles of Electrical Engineering I

The formulae related to Δ–Y equivalents are given below:

$$R_1 = \frac{R_b R_c}{R_a+R_b+R_c} \qquad R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1}$$

$$R_2 = \frac{R_a R_c}{R_a+R_b+R_c} \qquad R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2}$$

$$R_3 = \frac{R_a R_b}{R_a+R_b+R_c} \qquad R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3}$$



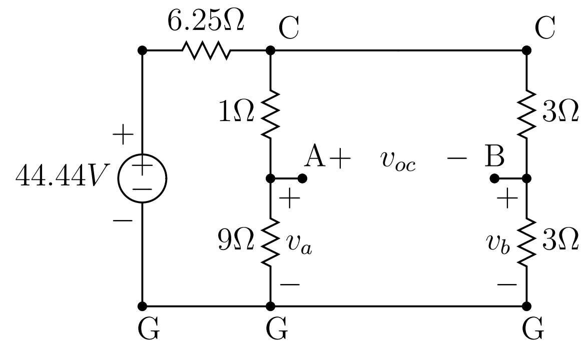


Figure 1

Problem: Our interest in this problem is to determine the Thevenin Equivalent circuit at the terminals A and B of the circuit shown in Figure 1. You can solve for open circuit voltage v_{oc} and short circuit current i_{sh} by different methods, including Node voltage method and mesh current method. The answers are $v_{oc} = 6.66$ V and $i_{sh} = 2.4$ A (see next page for the details of Node voltage method). In what follows, we will determine Thevenin resistance R_{Th} at the terminals A and B by constructing an independent source-less circuit as shown in Figure 2. For clarity, the circuit of Figure 2 is re-drawn as in Figure 3. In order to determine the resistance between the terminals A and B, we need a Δ to Y transformation. We can transform the Δ between the terminals A, C, and G (comprising of resistances 9Ω, 1Ω, and 6.25Ω), to a Y as shown in Figure 4. Here R_1, R_2, and R_3 are respectively given by

$$R_1 = \frac{9}{16.25}\Omega,\ R_2 = \frac{9(6.25)}{16.25}\Omega,\ R_3 = \frac{6.25}{16.25}\Omega.$$

Finally, from Figure 4, we can determine the resistance between the terminals A and B by series-parallel combinations as

$$R_{Th} = 2.775\Omega.$$

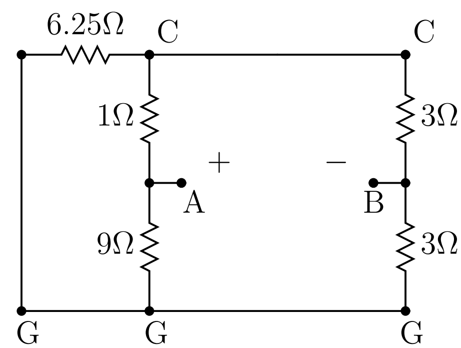


Figure 2

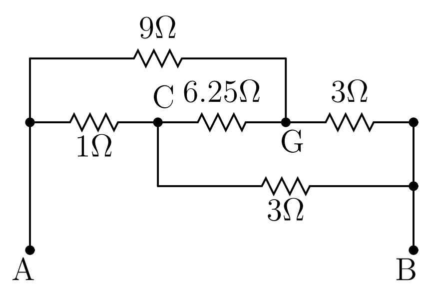


Figure 3

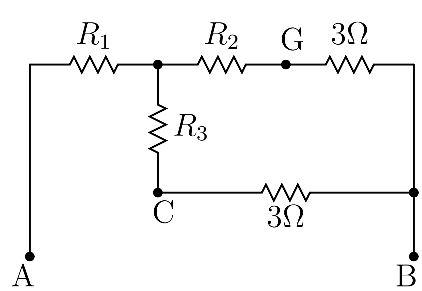


Figure 4

Determination of Open Circuit Voltage v_{oc} by Node voltage method: Consider the circuit of Figure 5 where node voltages v_s, v_a, v_b, and v_c are marked while taking node G as the reference node. We note that $v_s = 44.44$ V. We can easily write the following three node equations:

$$\begin{aligned} \frac{v_a}{9} + \frac{v_a - v_c}{1} &= 0 \\ \frac{v_b}{3} + \frac{v_b - v_c}{3} &= 0 \\ \frac{v_c - 44.44}{6.25} + \frac{v_c - v_a}{1} + \frac{v_c - v_b}{3} &= 0. \end{aligned}$$

By solving the above equations, we get $v_{oc} = v_a - v_b = 6.66$ V. In fact, we have $v_a = 15$ V, $v_b = 8.33$ V, and $v_c = 16.66$ V.

Solution:
$v_c = 16.66V,\ v_b = 8.33V,\ v_a = 15V$

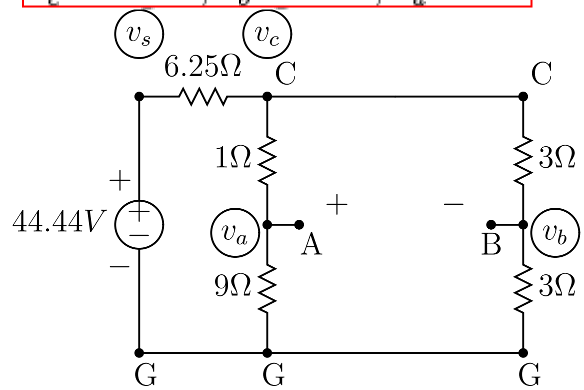


Figure 5

Determination of Short Circuit Current i_{sh} by Node voltage method: Consider the circuit of Figure 6 where node voltages v_s, v_a, v_b, and v_c are marked while taking node G as the reference node. We note that $v_s = 44.44$ V and $v_a = v_b$. We can easily write the following three node equations:

$$\begin{aligned} \frac{v_a}{9} + \frac{v_a - v_c}{1} + i_{sh} &= 0 \\ \frac{v_b}{3} + \frac{v_b - v_c}{3} - i_{sh} &= 0 \\ \frac{v_c - 44.44}{6.25} + \frac{v_c - v_a}{1} + \frac{v_c - v_b}{3} &= 0. \end{aligned}$$

Solution:
$v_c = 14.4V,\ v_a = v_b = 10.8V$

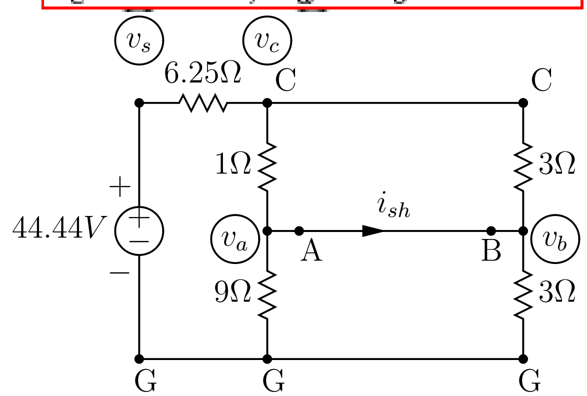
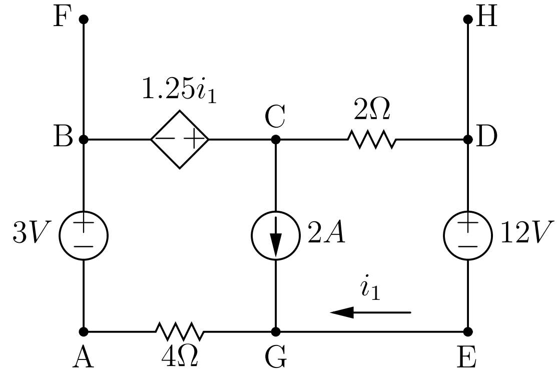


Figure 6

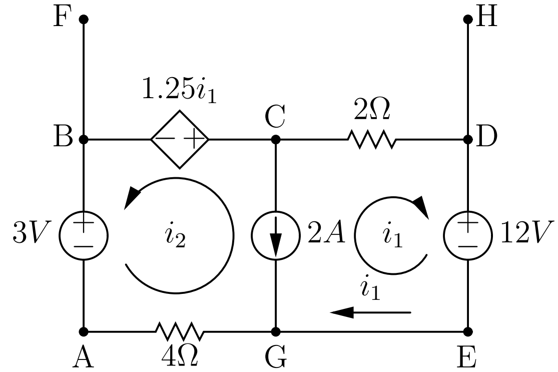
By solving the above equations, we get $v_a = v_b = 10.8$ V and $v_c = 14.4$ V. This yields $i_{sh} = 2.4$ A, and thus $R_{Th} = \frac{v_{oc}}{i_{sh}} = \frac{6.66}{2.4} = 2.775\Omega$.

Home-work (not collected): Determine Open Circuit Voltage v_{oc} and Short Circuit Current i_{sh} by Mesh Current Method.

Thevenin Equivalent Circuit – Example



We would like to determine the Thevenin equivalent circuit at the terminals F and H.



Determination of v_{oc} by Mesh current method: We select two mesh currents i_1, and i_2 as shown. The dependent voltage source $1.25i_1$ is controlled by i_1 which is one of the assumed mesh currents. We note also that the independent current source 2A is related to the assumed mesh currents by the equation,

$$i_1 + i_2 = -2.$$

Besides the above equation, we need one KVL equation to solve for the two unknowns i_1 and i_2. The KVL can be written for the loop ABCDEGA (Super mesh of ABCGA and GCDEG),

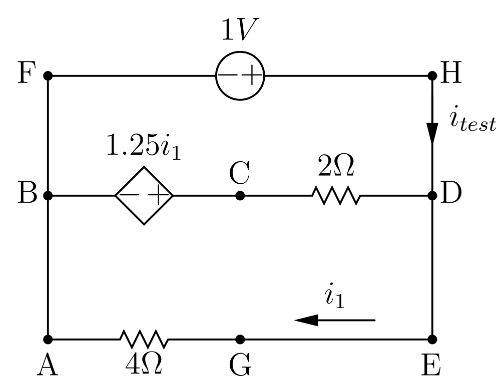
$$3{+}1.25i_1{-}i_1 2{-}12{+}i_2 4 = 0 \quad \Rightarrow \quad -0.75i_1{+}4i_2 = 9.$$

By solving the above equations, we get

$$i_1 = -\frac{17}{4.75} = -3.579A, \quad i_2 = 1.579A.$$

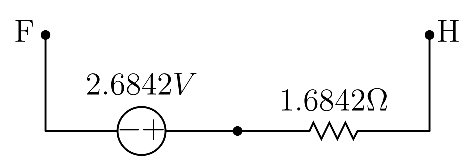
The voltage rise from F to H $v_{oc} = 1.25i_1 - 2i_1 = 2.6842V$.

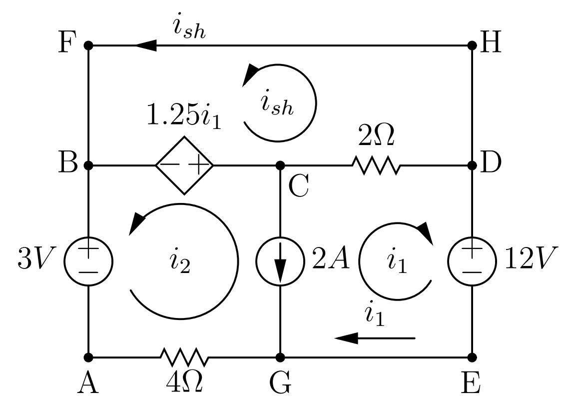
Determination of R_{Th} by using a test source: The independent voltage sources are set to zero, i.e a short is placed in their locations. The independent current source is set to zero, i.e. it is opened. The resulting circuit is shown on the right where a test voltage of 1 V between the terminals F and H is applied. It is easy to compute i_1 as 0.25A and current in 2Ω resistance from D to C as $0.34375A$. Hence $i_{test} = 0.59375A$. As such



$$R_{Th} = \frac{1}{0.59375} = 1.6842\Omega.$$

The Thevenin equivalent circuit is shown on the right.





Determination of i_{sh} by Mesh current method: In this example, one can easily determine i_{sh} by inspection. For example, by writing the KVL for the loop ABFHDEGA, we can first determine i_2. Then by writing KCL at node G, we can determine i_1. Once i_1 is known, we can determine i_{sh} by writing the KVL for the loop BCDHFB.

To use the Mesh current method systematically, we start by selecting three mesh currents i_1, i_2, and i_{sh} as shown. The dependent voltage source $1.25i_1$ is controlled by i_1 which is one of the assumed mesh currents. We note also that the independent current source 2A is related to the assumed mesh currents by the equation,

$$i_1 + i_2 = -2.$$

Besides the above equation, we need two KVL equations to solve for the three unknowns i_1, i_2, and i_{sh}. The KVL can be written for the loop ABFHDEGA (Super mesh of ABCGA, GCDEG, and FHDCBF),

$$3 - 12 + i_2 4 = 0.$$

Also, the KVL can be written for the mesh BCDHFB,

$$1.25i_1 - (i_1 + i_{sh})2 = 0.$$

By solving the above equations, we get $i_1 = -4.25$, $i_2 = 2.25$, and $i_{sh} = 1.59375$ A. We can now easily verify that

$$R_{Th} = \frac{v_{oc}}{i_{sh}} = 1.6842\Omega.$$

Principles of Electrical Engineering I

Example – Thevenin Equivalent Circuit

Fall 1999 quiz: Determine the Thevenin equivalent circuit of the circuit shown in Figure 1 with terminal b positive with respect to a.

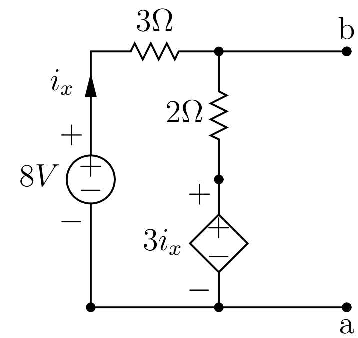


Figure 1

Node Voltage Method to determine v_{OC}: In Figure 2, pertinent node voltage variables are marked by selecting node a as the reference node. We can easily determine i_x as

$$i_x = \frac{8 - v_{OC}}{3}.$$

Also, we can write the node equation at node b as

$$\frac{v_{OC} - 3i_x}{2} + \frac{v_{OC} - 8}{3} = 0.$$

By solving the above two equations, we get

$$v_{Th} = v_{OC} = 5\,V.$$

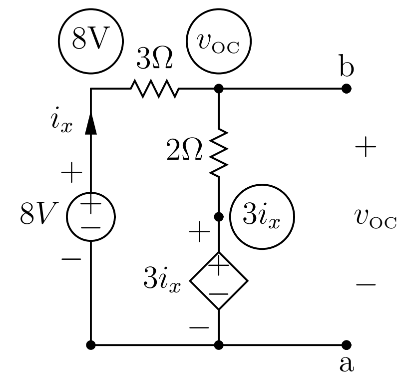


Figure 2

Node Voltage Method to determine i_{Sh}: In Figure 3, pertinent node voltage variables are marked by selecting node a as the reference node while shorting the nodes a and b. We can easily determine i_x as

$$i_x = \frac{8 - 0}{3} A.$$

Also, we can write the node equation at node b as

$$\frac{0 - 3i_x}{2} + \frac{0 - 8}{3} + i_{Sh} = 0.$$

By solving the above two equations, we get

$$i_{Sh} = \frac{20}{3}\,A.$$

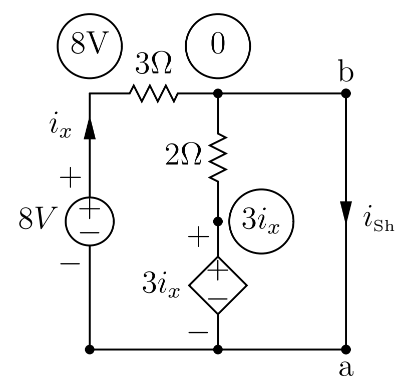
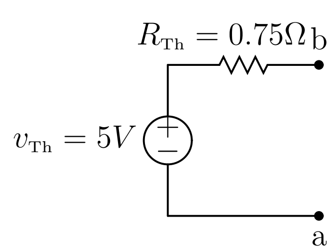


Figure 3

Then, the Thevenin resistance R_{TH} is given by

$$R_{TH} = \frac{v_{OC}}{i_{Sh}} = 0.75\Omega.$$



Thevenin Equivalent Circuit

Fall 1999 quiz: We determine next the Thevenin equivalent circuit of the circuit shown in Figure 1 while using Mesh Current method.

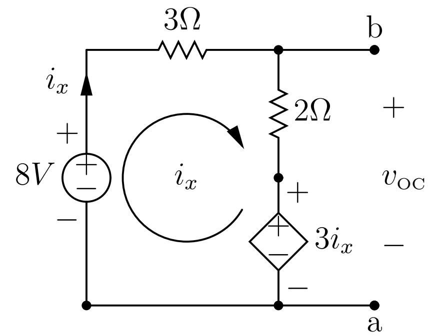


Figure 4

Mesh Current Method to determine $v_{\rm OC}$: In Figure 4, mesh current i_x is marked (Note that there is only one mesh). We can easily write the mesh equation as

$$8 - 3i_x - 2i_x - 3i_x = 0.$$

This yields $i_x = 1$ A. We note that

$$v_{\rm OC} = 3i_x + 2i_x = 5\,V.$$

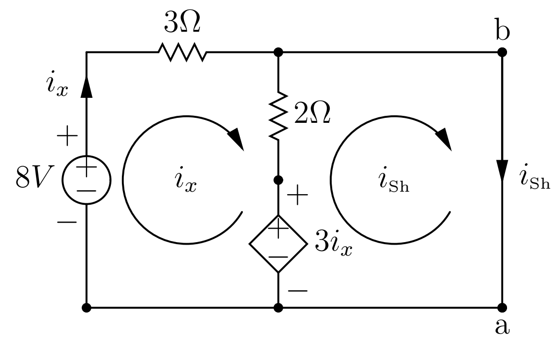


Figure 5

Mesh Current Method to determine $i_{\rm Sh}$: In Figure 5, Mesh Current variables are marked while shorting the nodes a and b (Note that there are now two meshes, and hence there are two mesh current variables, i_x and $i_{\rm Sh}$). We can easily write the two mesh equations as

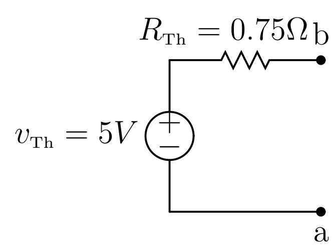
$$\begin{aligned} 8 - 3i_x - 2(i_x - i_{\rm Sh}) - 3i_x &= 0 \\ 3i_x - 2(i_{\rm Sh} - i_x) &= 0. \end{aligned}$$

By solving the above two equations, we get

$$i_x = \frac{8}{3}A \;\;\text{and}\;\; i_{\rm Sh} = \frac{20}{3}A.$$

Then, the Thevenin resistance $R_{\rm TH}$ is given by

$$R_{\rm TH} = \frac{v_{\rm OC}}{i_{\rm Sh}} = 0.75\Omega.$$



Thevenin Equivalent Circuit

Test Voltage and Test Current Method to determine $R_{\rm TH}$: Figure 6 shows the set up. We can easily write the two KCL equations as

$$\begin{aligned} v_t + 3i_x &= 0 \\ v_t - 2(i_x + i_t) - 3i_x &= 0. \end{aligned}$$

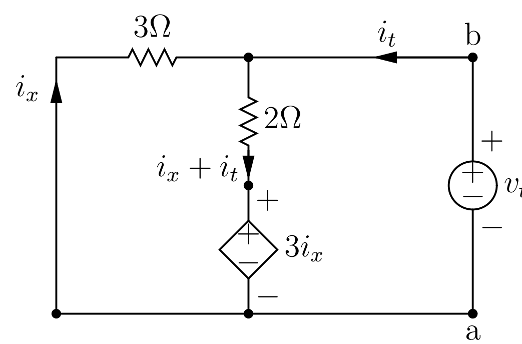


Figure 6

By solving the above two equations, we get

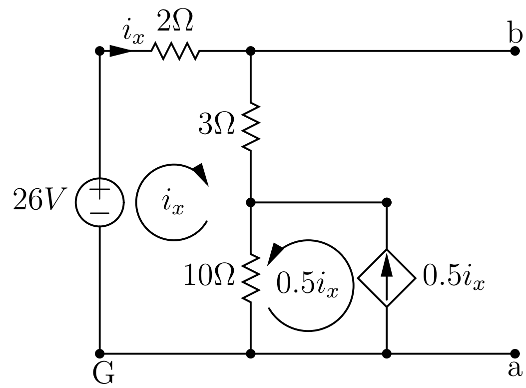
$$i_x = -\frac{v_t}{3} \;\;\text{and}\;\; i_t = \frac{4v_t}{3}.$$

Then, the Thevenin resistance $R_{\rm TH}$ is given by

$$R_{\rm TH} = \frac{v_t}{i_t} = \frac{3}{4} = 0.75\Omega.$$

Thevenin Equivalent Circuit – Example

We would like to determine the Thevenin equivalent circuit at the terminals a and b. To do so, we plan to compute all of the following three: (1) open circuit voltage, (2) short circuit current, and (3) Thevenin resistance R_{Th} directly.



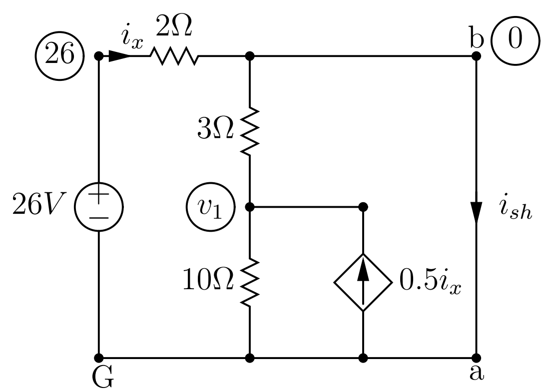
Determination of open circuit voltage v_{oc}: Using mesh current analysis, determine the open circuit voltage v_{oc} between the terminals a and b. Choose appropriate mesh currents in any direction you like. None of the methods other than mesh current analysis are acceptable.
There are two meshes and we chose i_x and $0.5i_x$ as mesh currents as shown.

One obvious KVL equation can be written as

$$26 = 2i_x + 3i_x + 10\left(\frac{3}{2}i_x\right).$$

This gives us $i_x = 1.3$ A. We can then compute

$$v_{oc} = 26 - 2i_x = 23.4V$$



Determination of short circuit current i_{sh}: Using nodal analysis, determine the short circuit current i_{sh} from the terminal a to b. Using G as the reference, choose appropriate node voltages you like. None of the methods other than nodal analysis are acceptable.
Using G as the reference, we can mark easily node voltages as shown. It is easy to compute

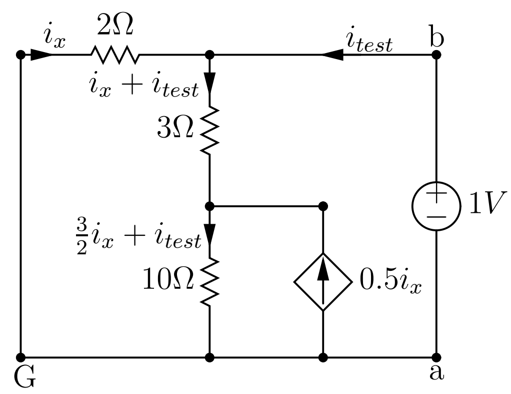
$$i_x = \frac{26}{2} = 13A.$$

Also, we can write the KCL equation at the node marked by v_1 as

$$\frac{v_1}{3} + \frac{v_1}{10} - 0.5i_x = 0 \quad \Rightarrow \quad v_1 = 15V.$$

By writing the KCL equation at the top node of 3Ω resistance, we get

$$i_{sh} = i_x + \frac{v_1}{3} \quad \Rightarrow \quad i_{sh} = 18A.$$



Direct determination of R_{Th} from the independent source-less network: The independent source-less network can be constructed as shown on the right side. Determine i_{test} by any method. Show all your work.
We can mark by inspection, the branch currents as shown. Also,

$$i_x = -\frac{1}{2}A.$$

Then, by writing one of the KVL equations, we get

$$1 = 3(i_x + i_{test}) + 10\left(\frac{3}{2}i_x + i_{test}\right) \quad \Rightarrow \quad i_{test} = 0.7692A$$

Determine the Thevenin resistance R_{Th} as

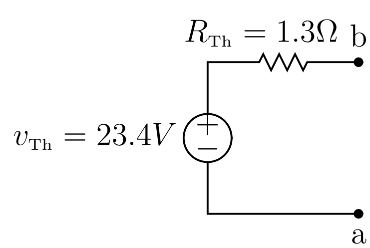
$$R_{Th} = \frac{1}{i_{test}} = \frac{1}{0.7692} = 1.3\Omega.$$

The value for R_{Th} as calculated from $\frac{v_{oc}}{i_{sh}}$ is

$$R_{Th} = \frac{v_{oc}}{i_{sh}} = \frac{23.4}{18} = 1.3\Omega.$$

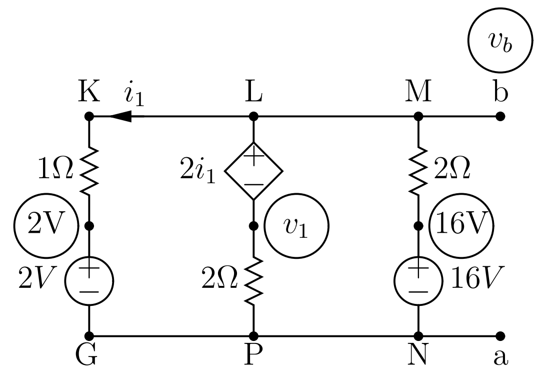
Yes, both ways of computing R_{Th} agree with each other.

Mark appropriate values for the Thevenin equivalent circuit shown on the right.



Thevenin Equivalent Circuit – Example

We would like to determine the Thevenin equivalent circuit at the terminals a and b. To do so, we need to compute open circuit voltage and short circuit current.



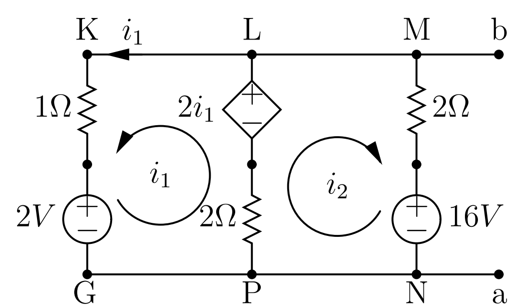
Determination of v_{oc} by Node voltage method: Using G as the reference, we mark the node voltages as shown. Here v_b and v_1 are unknown. We note that $v_b = v_{oc}$. We need to express the controlling current i_1 in terms of the node voltages. We see easily that $i_1 = v_b - 2$. Also, we note that $v_1 = v_b - 2i_1 = v_b - 2(v_b - 2) = 4 - v_b$. We can now easily write the node equation at the nodes K, L, and M (all of them as one node) as

$$\frac{v_b - 16}{2} + \frac{v_1}{2} + \frac{v_b - 2}{1} = 0.$$

We can substitute for v_1 as $4 - v_b$, and rewrite the above equation as

$$\frac{v_b - 16}{2} + \frac{4 - v_b}{2} + \frac{v_b - 2}{1} = 0.$$

By solving which we get $v_b = v_{oc} = 8$V.



Determination of v_{oc} by Mesh current method: We select two mesh currents i_1 and i_2 as shown. The dependent voltage source $2i_1$ is controlled by i_1 which is one of the assumed mesh currents. To solve for the two currents i_1 and i_2, we need to write two KVL equations. The KVL for the loop LKGPL can be written as

$$-i_1 - 2 - 2(i_1 + i_2) + 2i_1 = 0 \quad \Rightarrow \quad i_1 + 2i_2 = -2.$$

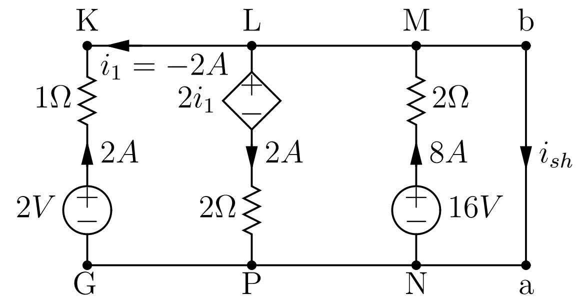
Similarly, the KVL for the loop MNPLM can be written as

$$-2i_2 - 16 - 2(i_1 + i_2) + 2i_1 = 0 \quad \Rightarrow \quad 4i_2 = -16.$$

From the second equation, we get $i_2 = -4$A. Substituting this in the first equation, we get $i_1 = 6$A. Knowing $i_2 = -4$A, we get

$$v_{oc} = 16 + 2i_2 = 8v.$$

Determination of short circuit current i_{sh}: The short circuit current i_{sh} can be easily calculated by inspection. (You can try other methods to compute i_{sh}.) To do so, at first, we calculate the current in each branch. By writing the loop equation GKLMbaNPG, we see the current from G to K as 2 A. Similarly, by writing the loop equation LMbaNPL, we see the current from L to P as 2 A. Finally, by writing the loop equation MbaNM, we see the current from N to M as 8 A. We observe $i_{sh} = 8 + 2 - 2 = 8$ A.

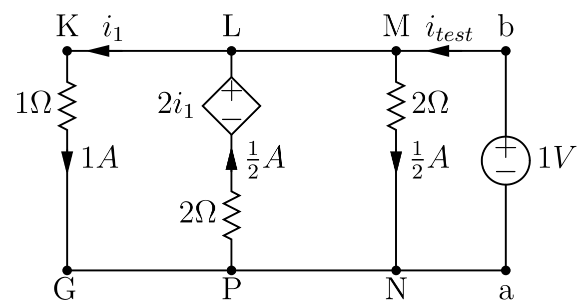


The Thevenin resistance R_{Th} is given by

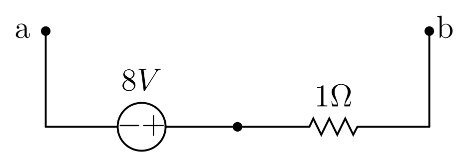
$$R_{Th} = \frac{v_{oc}}{i_{sh}} = \frac{8}{8} = 1\Omega.$$

Determination of R_{Th} by using a test source: The independent voltage sources are set to zero, i.e a short is placed in their locations. The resulting circuit is shown on the right where a test voltage of 1 V between the terminals F and H is applied. It is easy to compute i_1 as 1A. Knowing i_1, we see easily that the current from P to L as $\frac{1}{2}$ A. Also, we see the current from M to N as $\frac{1}{2}$ A. Then, writing the KCL equation at the nodes K, L, and M (all of them as one node), we see $i_{test} = 1$ A. This implies that

$$R_{Th} = \frac{1}{1} = 1\Omega.$$



The Thevenin equivalent circuit is shown on the right.



Principles of Electrical Engineering I

Example 1: The purpose of this example is to show that v_{OC} is zero whenever there are no independent sources in the given circuit, and to illustrate that R_{Th} could be negative whenever active elements are present.

Determine the open circuit voltage v_{OC} at the terminals a and b for the circuit of Figure 1a.

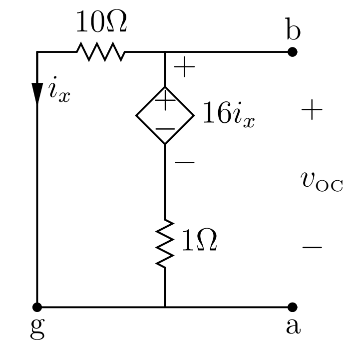


Figure 1a

Determination of v_{OC}: To determine v_{OC}, take g as the reference node and mark the node voltages as shown in Figure 1b. Obviously,

$$i_x = \frac{v_{\text{OC}}}{10}, \quad \text{and} \quad v_1 = v_{\text{OC}} - 16i_x = -0.6v_{\text{OC}}.$$

Thus,

$$i_1 = \frac{v_1}{1} = -0.6v_{\text{OC}}$$

Now the node equation at b or a implies that

$$i_x + i_1 = 0 \Rightarrow \frac{v_{\text{OC}}}{10} - 0.6v_{\text{OC}} = -0.5v_{\text{OC}} = 0.$$

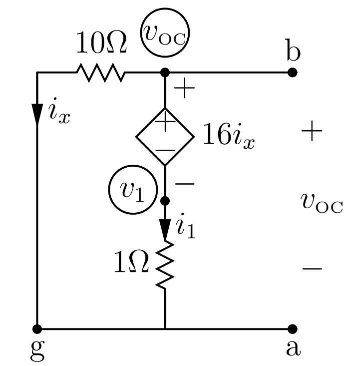


Figure 1b

The last equation certainly implies that $v_{\text{OC}} = 0$.

Determination of R_{Th}: To determine R_{Th}, we use test voltage and test current method. We apply the voltage v_t to the circuit, and compute i_t drawn by the circuit. Thus, consider Figure 1c. As before we note that

$$i_x = \frac{v_t}{10}, \quad \text{and} \quad v_1 = v_t - 16i_x = -0.6v_t.$$

Thus,

$$i_1 = \frac{v_1}{1} = -0.6v_t$$

Now the node equation at b or a implies that

$$i_t = i_x + i_1 \Rightarrow i_t = \frac{v_t}{10} - 0.6v_t = -0.5v_t.$$

Thus the Thevenin resistance R_{Th} equals $\frac{v_t}{i_t} = -2\Omega$.

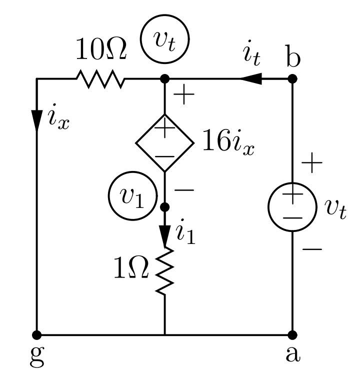


Figure 1c

Example 2. Determine the Thevenin equivalent circuit of two resistances R_1 and R_2 connected in parallel. Obviously, as there are no independent sources, the open circuit voltage is zero, and $R_{\text{TH}} = \frac{R_1R_2}{R_1+R_2}$.

Example 3. Determine the Thevenin equivalent circuit of an ideal battery of 12 V. Obviously, the open circuit voltage is 12 V, and $R_{\text{TH}} = 0$.

Principles of Electrical Engineering I
Maximum Power Transfer

Consider the interconnection of two circuits α and β as shown in Figure 1. The circuit α is an active circuit with sources (both independent and dependent) while the circuit β is a passive load on the circuit α. Let the equivalent resistance of circuit β be R_L. Determine R_L such that power transferred by the circuit α to the load R_L is maximum.

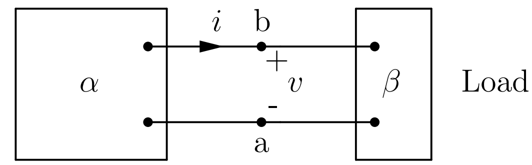


Figure 1

By replacing the circuit α by its Thevenin equivalent, we can redraw the circuit of Figure 1 as that in Figure 2.

Equivalent of Circuit α

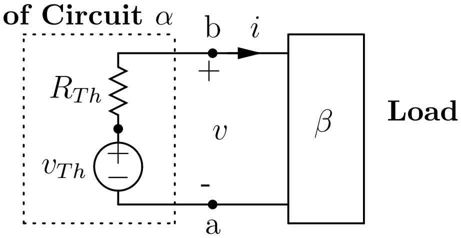


Figure 2

Current i is given by

$$i = \frac{v_{Th}}{R_{Th} + R_L}$$

Power consumed by R_L,

$$P = i^2 R_L = \frac{v_{Th}^2 R_L}{(R_{Th} + R_L)^2}$$

We need to determine R_L such that P is maximum. We need to set

$$\frac{dP}{dR_L} = 0.$$

Differentiation formula,

$$d\Big[V^{-1}U\Big] = V^{-1}\,dU - V^{-2}dV\,U = \frac{V\,dU - dV\,U}{V^2}.$$

Applying the above rule with $V = (R_{Th} + R_L)^2$ and $U = R_L$, we get

$$\frac{\partial P}{\partial R_L} = \frac{v_{Th}^2\left[(R_{Th} + R_L)^2 - 2(R_{Th} + R_L)R_L\right]}{(R_{Th} + R_L)^4} = 0$$

$$\Rightarrow \quad (R_{Th} + R_L)^2 - 2(R_{Th} + R_L)R_L = 0 \quad \Rightarrow \quad (R_{Th} + R_L) - 2R_L = 0.$$

This yields

$$R_L = R_{Th}.$$

In words, the load resistance R_L equals the Thevenin resistance (R_{Th}) in order to maximize the power P consumed by the load.

Second derivative of P with respect to R_L confirms that P is maximum when $R_L = R_{Th}$. Verify yourself.

$$\text{Maximum Power} = \frac{R_{Th}}{(2R_{Th})^2}v_{Th}^2 = \frac{v_{Th}^2}{4R_{Th}}.$$

HW from Nilsson and Riedel 8th and 9th editions

Set up the equations for the following problems. Solve them numerically by utilizing your calculator or Matlab.

Nilsson and Riedel 8th edition:

4.61, 4.62

4.72 You can first develop a Thevenin equivalent to the circuit left of $50k\Omega$, once you do that replace the circuit left of $50k\Omega$ by its Thevenin equivalent and then develop a Thevenin equivalent at the terminals a and b.

4.77 There are no independent sources. So, Thevenin voltage is zero. Find the Thevenin resistance by test voltage test current method.

Nilsson and Riedel 9th edition:

4.60, 4.62

4.73 You can first develop a Thevenin equivalent to the circuit left of $50k\Omega$, once you do that replace the circuit left of $50k\Omega$ by its Thevenin equivalent and then develop a Thevenin equivalent at the terminals a and b.

4.78There are no independent sources. So, Thevenin voltage is zero. Find the Thevenin resistance by test voltage test current method.

An email will inform the due date

Name in CAPITAL LETTERS:
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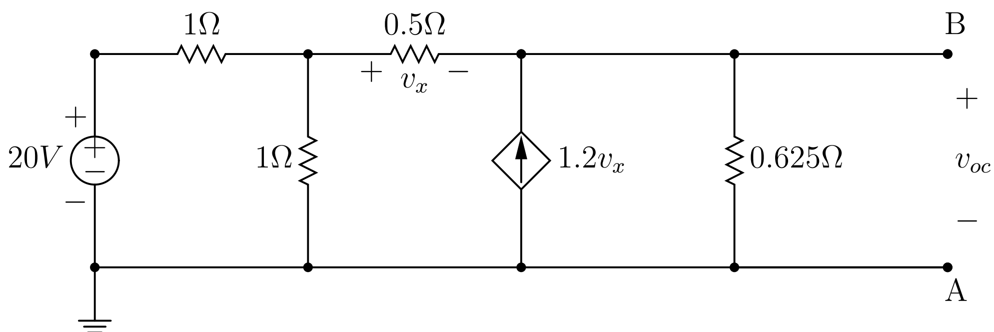
HW: Thevenin Equivalent Circuit, 5 pages

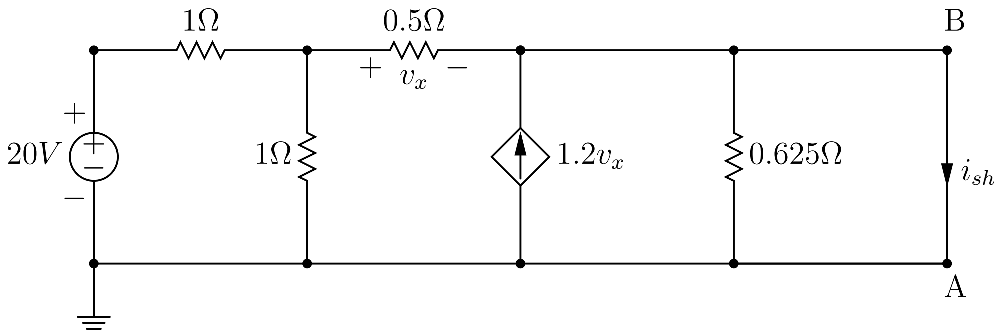
332:221 Principles of Electrical Engineering I

This problem will be collected and graded.

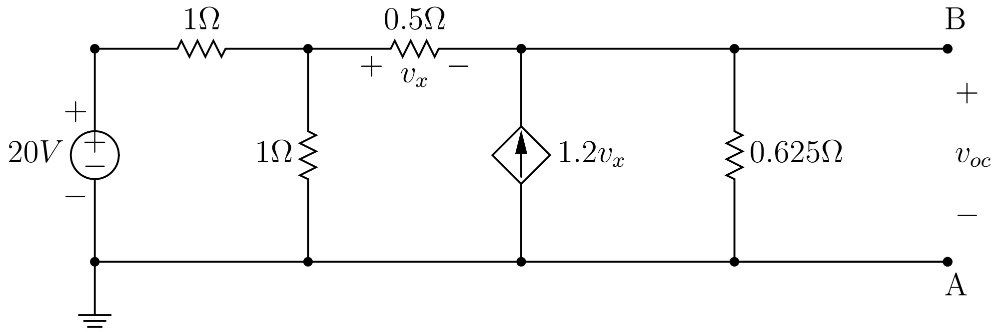
(Problem 1a) Determination of $v_{Th} = v_{oc}$ by node voltage method: Consider the circuit shown below. Our aim in this problem is to determine the open circuit voltage v_{oc} at the terminals A and B by utilizing the node voltage method. Consider the grounded terminal as the reference node.



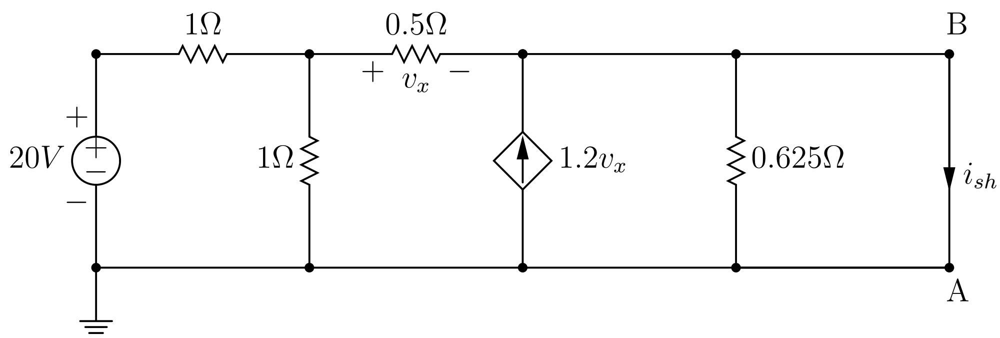
(Problem 1b) Determination of i_{sh} by node voltage method: Consider the circuit shown below. Our aim in this problem is to determine the short circuit current i_{sh} through the terminals A and B by utilizing the node voltage method. Consider the grounded terminal as the reference node.



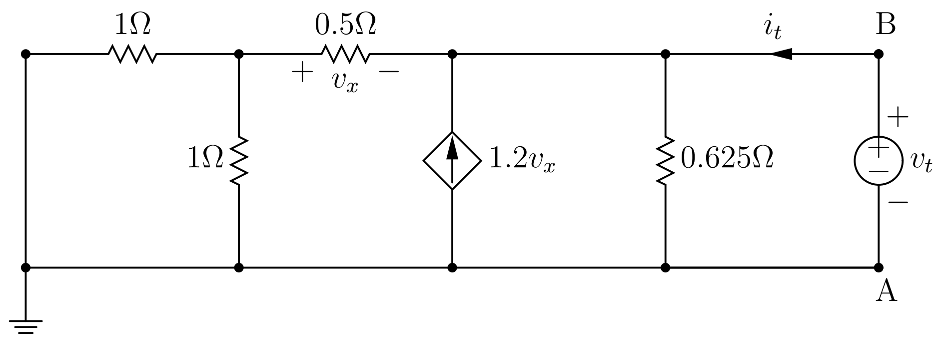
(Problem 1c) Determination of $v_{Th} = v_{oc}$ by mesh current method: Consider the circuit shown below. Our aim in this problem is to determine the open circuit voltage v_{oc} at the terminals A and B by utilizing the mesh current method.



(Problem 1d) Determination of i_{sh} by mesh current method: Consider the circuit shown below. Our aim in this problem is to determine the short circuit current i_{sh} through the terminals A and B by utilizing the mesh current method.



(Problem 1e) Determination of R_{Th} by test voltage and test current method: We first construct an independent source-less circuit. Apply a test voltage v_t at the terminals A and B, and determine the test current i_t supplied by it to the circuit. Then we recognize that the Thevenin resistance R_{Th} is the ratio $\frac{v_t}{i_t}$. Utilize any method to determine the ratio $\frac{v_t}{i_t}$. The independent source-less circuit along with the test source is as shown below.



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HW: DAC Thevenin Equivalent

332:221 Principles of Electrical Engineering I

Theme Example – DAC– Thevenin equivalent circuit at the output terminals HW, collected and graded

Digital to analog converter (DAC) circuit.

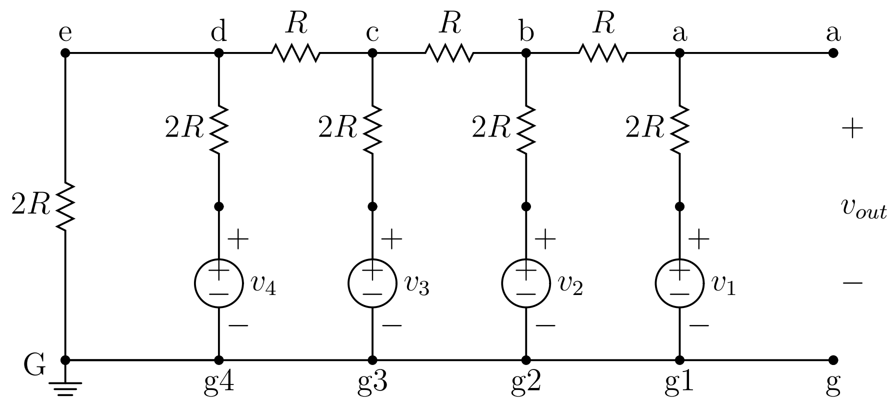


Figure 1

Determination of the Thevenin equivalent circuit at the output terminals 'g' and 'a':

Thevenin voltage v_{Th} is the open circuit voltage at the output terminals 'g' and 'a'. That is, it is the output voltage v_{out} when no load is connected at the terminals 'g' and 'a' as in Figure 1. Such a voltage has been determined as (you need not show this),

$$v_{Th} \;=\; \tfrac{1}{2}v_1 \;+\; \tfrac{1}{4}v_2 \;+\; \tfrac{1}{8}v_3 \;+\; \tfrac{1}{16}v_4.$$

Thevenin resistance R_{Th} is the same as the resistance R_{Th} seen from the output terminals 'g' and 'a' as shown in Figure 1R where all the source voltages v_1, v_2, v_3, and v_4 are set to zero. Determine Thevenin resistance R_{Th}.

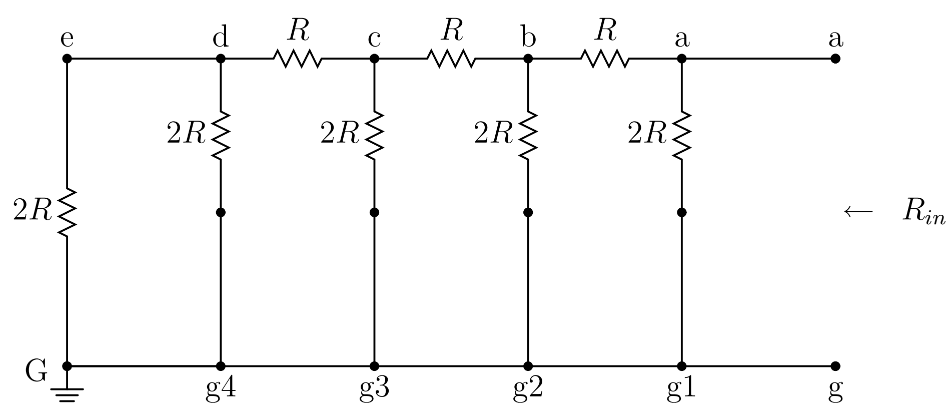
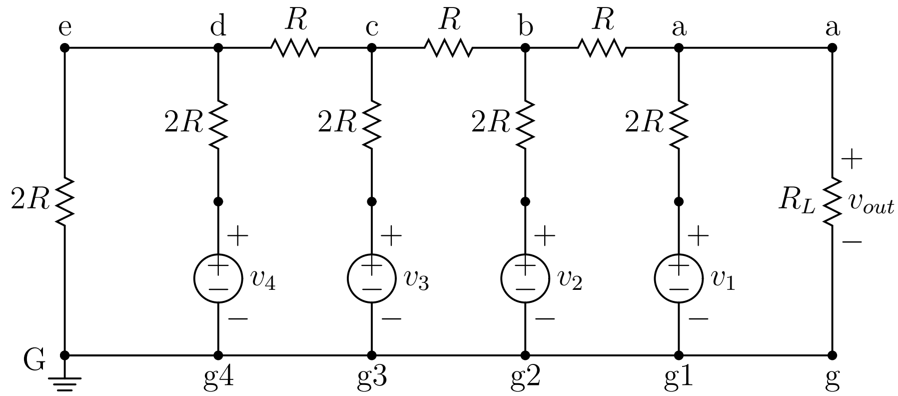


Figure 1R

Draw below the Thevenin equivalent circuit at the output terminals 'g' and 'a'.

Determination of v_{out} when a load R_L is connected at the output terminals 'g' and 'a': Connect a load of R_L at the output terminals 'g' and 'a' as shown below.



Modify the Thevenin equivalent circuit drawn earlier to include the load, then determine the resulting output voltage v_{out}. Note that v_{out} is a function of R_L. This implies that the load affects the output voltage v_{out}. **We will see later on how to avoid this.**