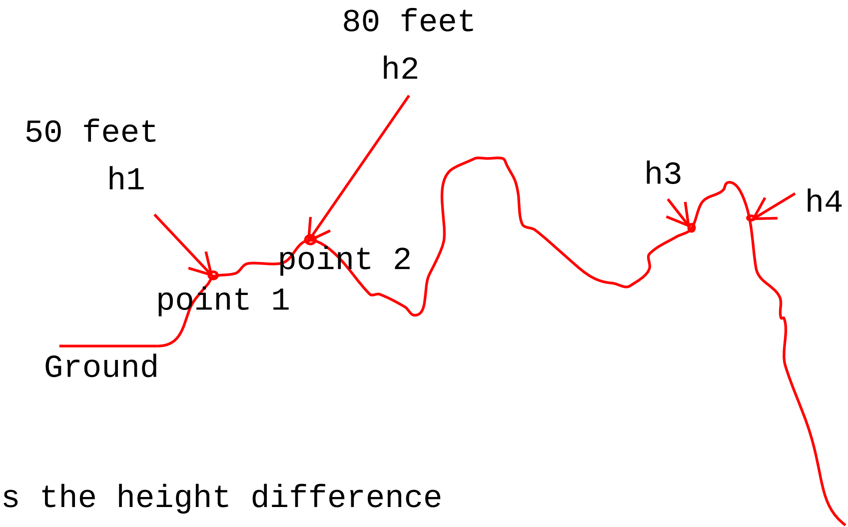
Gravitational hill
Gravitational potentials



What is the height difference
between points 1 and 2 ?

Which one is higher than which?

Suppose I do not know numerical
values, I know only symbols.
How can we indicate the height
difference?

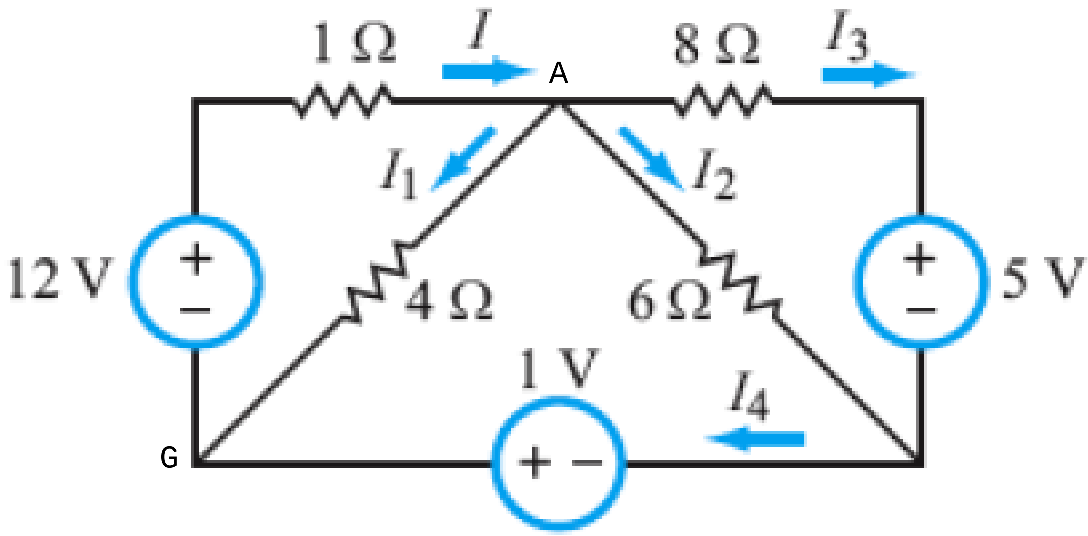
h1-h2 This is the height difference.
Which one is higher than which?

h2-h1 This is the height difference.
Which one is higher than which?

The one you start with is higher
than the other.

Motivating Example to introduce Node Voltage Method
and Mesh Current Method

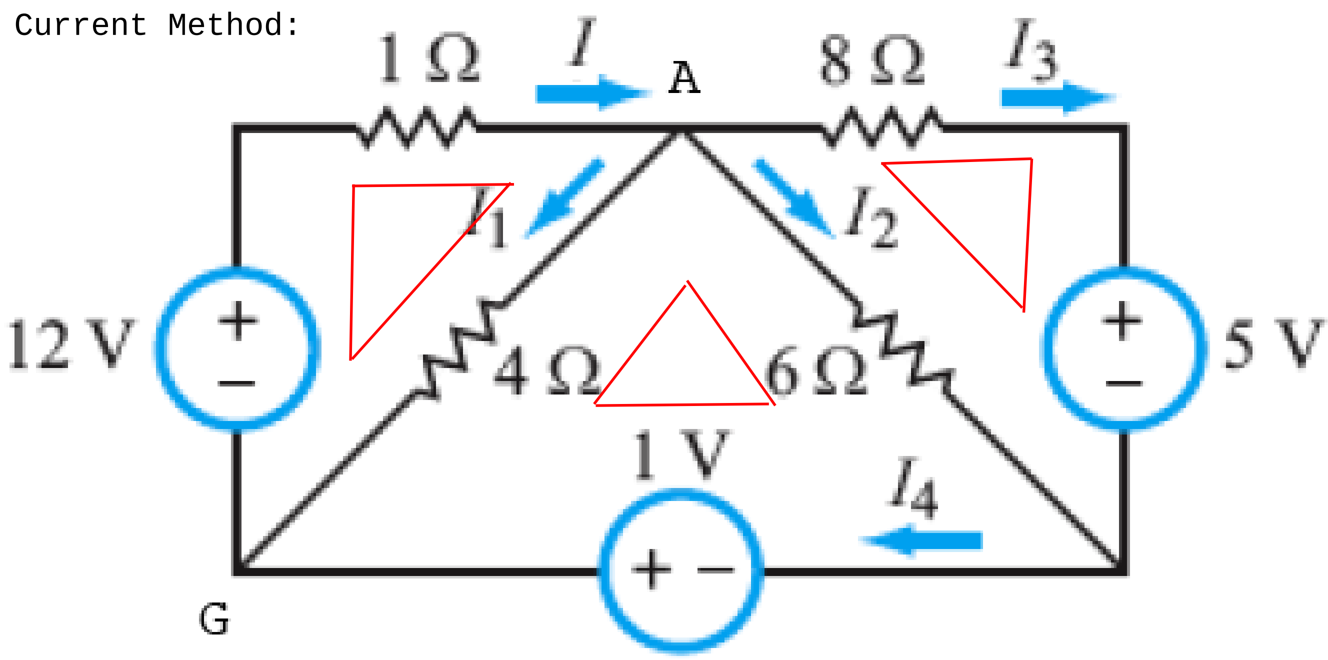
Node Voltage Method:



Take G as a reference node. What is the voltage of each other node with respect to G? Only voltage of node A is unknown. All others are known.

To solve for the unknown node voltages, we need to write as many KCL equations as there are unknowns.

Mesh Current Method:



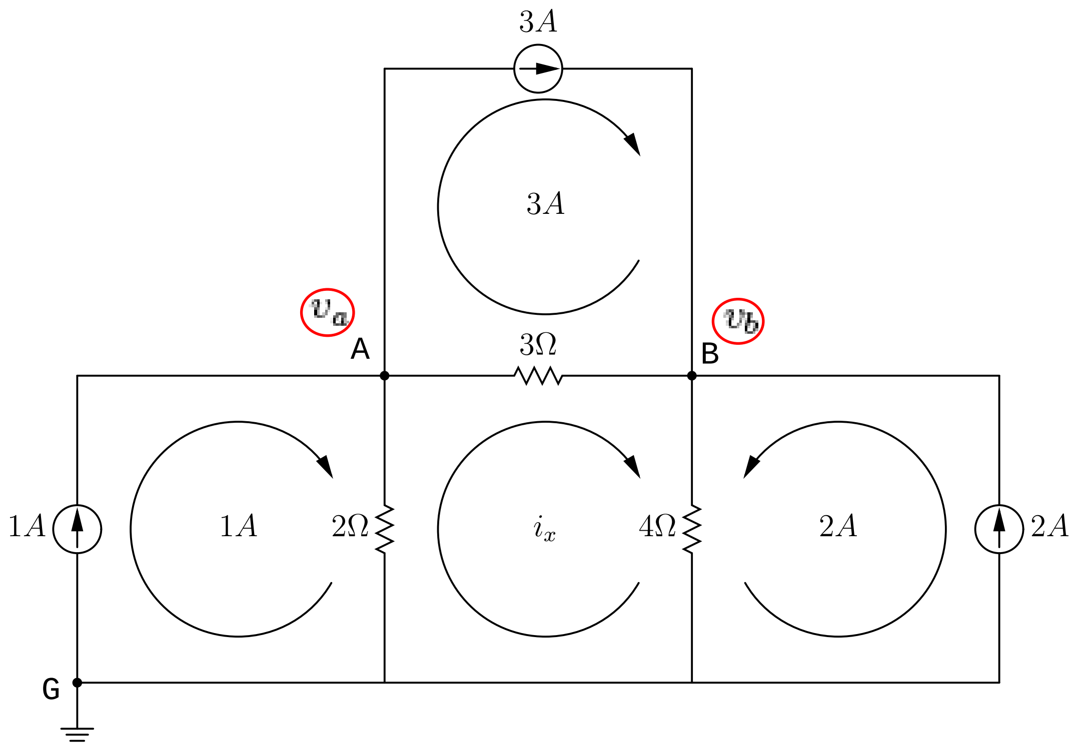
There are three meshes. Each mesh is a window. Each window has branches as its frame. We assume a mesh current flows around the window frame. Those branches which form frames for different meshes have two different mesh currents flowing through them. The resultant current equals the algebraic (signs taken into account) sum of two mesh currents. In the above circuit, 4 ohm branch and 6 ohm branch, each has two different mesh currents.

Some mesh currents could be known ahead and others unknown. To solve for the unknown currents, we need to write as many KVL equations as there are unknowns.

332:221 Principles of Electrical Engineering I

Mesh Current Method

Select the mesh currents as shown. Write an appropriate mesh equation (KVL equation) and solve for the unknown value i_x.



Solution: The middle mesh equation in the lower row of meshes is given by

$$2(i_x - 1) + 3(i_x - 3) + 4(i_x + 2) = 0 \quad \Rightarrow \quad i_x = \frac{1}{3}\,A.$$

Knowing i_x, we can determine any current or voltage in the given circuit. For example, the current in 3Ω resistance from B to A is $3 - i_x = \frac{8}{3}\,A$.

Node Voltage Method

One requires to introduce two unknown node voltages, to solve this circuit by Node Voltage Method. Select G as the reference node, and assign two unknown node voltages v_a and v_b to nodes at A and B. Then, we can write the following equations:

$$\frac{v_a}{2} + \frac{v_a - v_b}{3} + 3 - 1 = 0,$$

$$\frac{v_b}{4} + \frac{v_b - v_a}{3} - 3 - 2 = 0,$$

By solving the above equations, we get $v_a = \frac{4}{3}$ V and $v_b = \frac{28}{3}$ V. Knowing v_a and v_b, we can determine any current or voltage in the given circuit. For example, the current in 3Ω resistance from B to A is $\frac{v_b - v_a}{3} = \frac{8}{3}\,A$.

332:221 Principles of Electrical Engineering I
Introduction to Node Voltage Method & Mesh Current Method

Determination of current through a resistance connected between two nodes: Note that any given branch has two terminals, say 'a' and 'b' . Let the voltage of node 'a' with respect to a reference node G be v_a. Similarly, let the voltage of node 'b' with respect to the same reference node G be v_b. This is illustrated in Figure node-ab. Here $v_a - v_b$ is the voltage across the branch with node 'a' being the positive side and node 'b' being the negative side. Equivalently, $v_b - v_a$ is the voltage across the branch with node 'b' being the positive side and node 'a' being the negative side. Let the branch be a resistor with its resistance being R ohms.

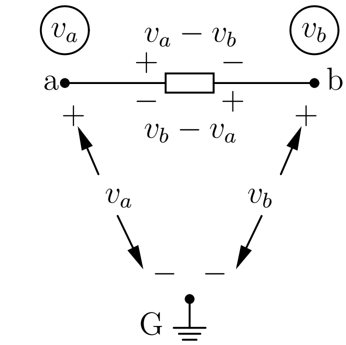
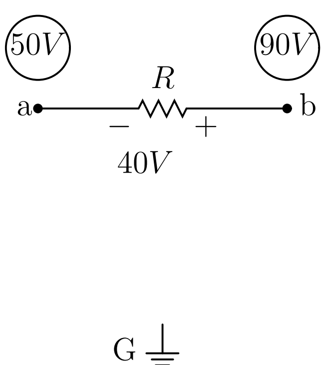


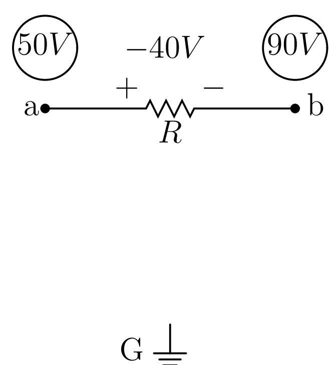
Figure node-ab

Then,
the current leaving node 'a' towards node 'b' is $\frac{v_a - v_b}{R}$.
Equivalently,
the current leaving node 'b' towards node 'a' is $\frac{v_b - v_a}{R}$.

Note: If the branch connected between nodes 'a' and 'b' is a voltage source, then v_a and v_b are related and the difference between them is the value of voltage source either it is an independent or dependent voltage source. **The current in the voltage source cannot be calculated easily in terms of its node voltages alone.** We need to determine the currents of other branches before we determine the current in a voltage source.

Numerical illustration:





Voltage of b with respect to a is
$v_b - v_a = 40V$
Current from b to a is $\frac{v_b - v_a}{R} = \frac{40}{R}$

Voltage of a with respect to b is
$v_a - v_b = -40V$
Current from a to b is $\frac{v_a - v_b}{R} = \frac{-40}{R}$

Both of the above figures are equivalent.
To determine the current leaving node b towards node a, we use $\frac{v_b - v_a}{R}$.
In exactly the same way,
to determine the current leaving node a towards node b, we use $\frac{v_a - v_b}{R}$.
Observe this carefully.

Example 1: Solve the circuit shown below in Figure 1a.
We can solve the circuit by series parallel combination of resistances and thus simplifying the circuit successively as shown in Figures 1b and 1c. In view of Figure 1c, it is easy to determine the current supplied by the 70 V source as 2 A. Then moving back to Figure 1b, one can determine the currents in 60Ω, 15Ω, and 5Ω resistances either by current division rule or otherwise. The entire solution is shown in Figure 1d.

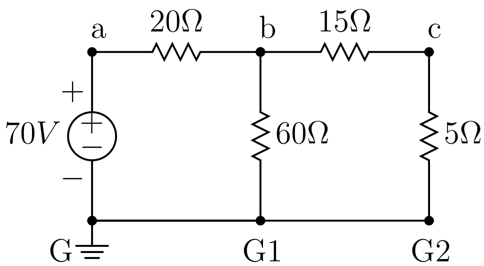


Figure 1a

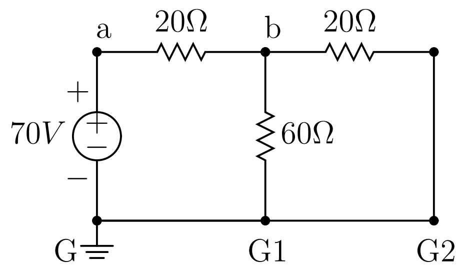


Figure 1b

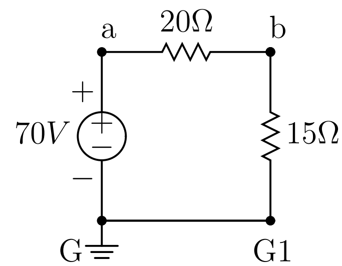


Figure 1c

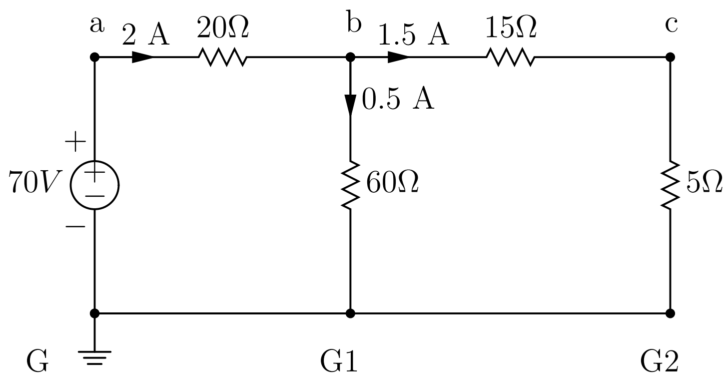


Figure 1d

Example 1 by Node Voltage Method: In Node Voltage Method method, we first select a reference node and then denote the voltage of each of other nodes by a variable as shown in Figure 1n. Here v_a shown in circle next to node 'a' is the voltage of node 'a' with respect to node 'G' and, similarly v_b shown in circle next to node 'b' is the voltage of node 'b' with respect to node 'G'. We observe easily that $v_a = 70$ V. In order to determine v_b, we first determine the currents of all branches connected to node 'b' in terms of the assigned node variables v_a and v_b. In doing so, we determine all currents either leaving the node or entering the node, and then, in view of KCL, add all of them and equate the sum to zero.

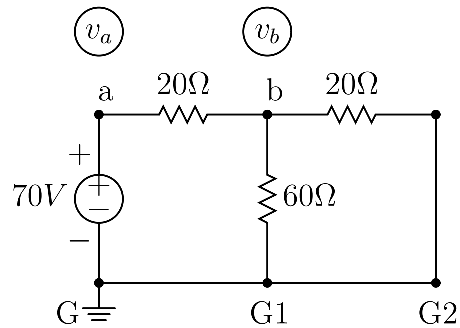


Figure 1n

The current leaving node 'b' towards node 'a' via 20Ω resistance is $\frac{v_b - v_a}{20} = \frac{v_b - 70}{20}$.

The current leaving node 'b' towards node 'G2' via 20Ω resistance is $\frac{v_b - 0}{20}$.

The current leaving node 'b' towards node 'G1' via 60Ω resistance is $\frac{v_b - 0}{60}$.

Thus, the KCL equation at node 'b' can be written as

$$\frac{v_b - 70}{20} + \frac{v_b}{20} + \frac{v_b}{60} = 0.$$

The solution of above equation yields $v_b = 30$ V. Once v_b is known, all the branch currents can be easily computed and are as given in Figure 1d in previous page.

Example 1 by Mesh Current Method: Mesh is a closed loop and forms a window in a planar circuit (a mesh is a loop without enclosing any other loops). In any given circuit with B branches and N nodes, there are $B-(N-1)$ meshes. For the circuit shown in Figure 1m, there are two meshes. In Mesh Current Method, a mesh current is assumed to circulate through each frame of the mesh. Each mesh has its own circulating current. Figure 1m which has two meshes has two circulating currents marked i_1 and i_2. The direction of mesh currents is chosen arbitrarily. In order to determine i_1 and i_2, we first determine the voltage across each resistance in terms of them. This can be done mentally as illustrated soon. Then, we write a KVL equation for each mesh. For the circuit shown in Figure 1m, the KVL equations are as follows:

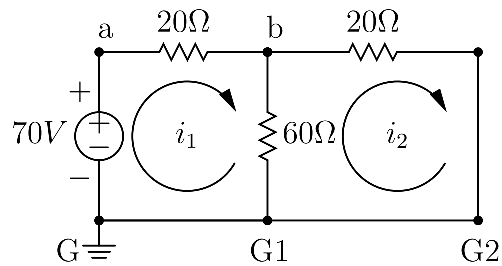


Figure 1m

$$70 - 20i_1 - 60(i_1 - i_2) = 0 \text{ for mesh 1,}$$

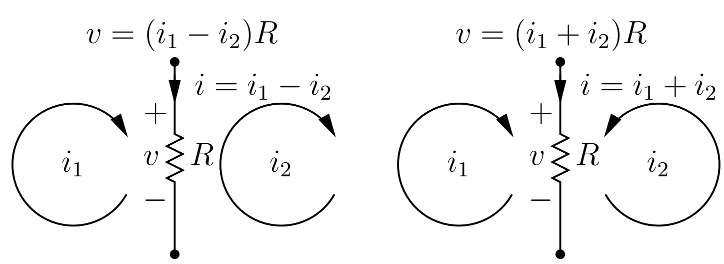
$$-20i_2 - 60(i_2 - i_1) = 0 \text{ for mesh 2.}$$

The solution of the above equations yields $i_1 = 2$ A and $i_2 = 1.5$ A. Once i_1 and i_2 are known, all the branch currents can be easily computed and are as given in Figure 1d in an earlier page where we solved the circuit by series parallel combinations.

Determination of voltage across a resistance in terms of mesh currents: Some branches carry only one mesh current if these branches are part of only one mesh and not parts of other meshes. In the circuit of Figure 1m, all branches except 60Ω resistance has only one mesh current. The voltage across such branches except sources can easily obtained by Ohm's law. However, 60Ω resistance has two mesh currents. **The resultant current in it is the algebraic sum (with appropriate signs taken into account)**. Once, the resultant current in a chosen direction is determined, the voltage across the resistance with its appropriate sign is given by Ohm's law.

Illustration of determining the resultant current through a branch common to two meshes:

If a particular branch is a resistance (impedance in general), its voltage can be easily calculated in terms of its mesh currents as illustrated on the right where $v = Ri$ in accordance with Ohm's law. Such a calculation can be done by inspection when we write the needed KVL mesh equations.



Note: Either an independent or dependent current source in a mesh yields an equation relating its value to the mesh current or currents passing through it. However, **the voltage across a current source cannot be calculated easily in terms of its mesh currents alone**. We need to determine the voltages of other branches before determining the voltage of current sources. This aspect is illustrated in other examples.

Example 2: Solve the circuit shown below in Figure 2a.
We can solve the circuit by series parallel combination of resistances and thus simplifying the circuit successively and then solving each simplified circuit, the results of which can then be interpreted for the given circuit in Figure 2a. Such results are shown in the circuit of Figure 2b.

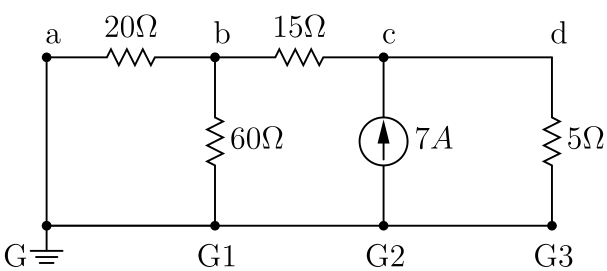


Figure 2a

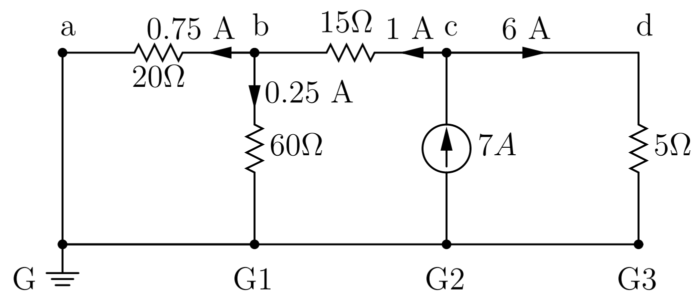


Figure 2b

Example 2 by Node Voltage Method: In order to solve the circuit shown in Figure 2a by **node voltage method**, the voltage of each node with respect to the reference G is marked in a circle as shown in the circuit of Figure 2c.

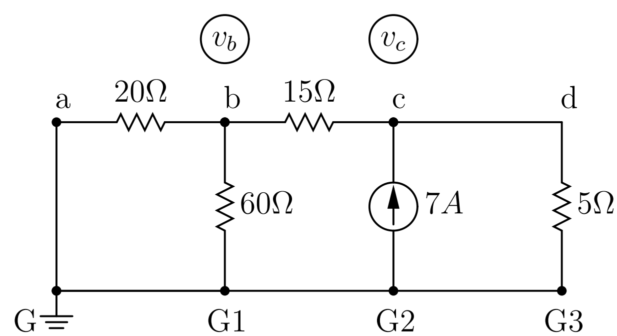


Figure 2c

By summing all the currents leaving the node 'b' and equating the sum to zero, we get

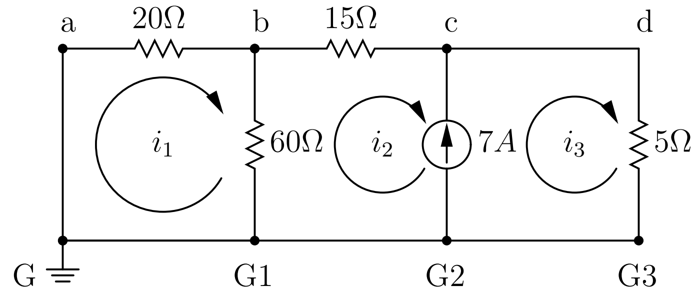
$$\frac{v_b - 0}{20} + \frac{v_b - 0}{60} + \frac{v_b - v_c}{15} = 0.$$

Similarly, the KCL equation at node 'c' can be written as

$$\frac{v_c - v_b}{15} + \frac{v_c - 0}{5} - 7 = 0.$$

The solution of the above equations yields $v_b = 15$ V and $v_c = 30$ V. Once v_b and v_c are known, all the branch currents can be easily computed and are as given in the circuit of Figure 2b.

Example 2 by Mesh Current Method: The circuit of Figure 2a has three meshes, and hence has three circulating currents i_1, i_2, and i_3 as marked. The direction of mesh currents is chosen arbitrarily.



We can relate the current source to the chosen mesh currents. We note that there are two mesh currents flowing through the current source, i_2 flows down from node 'c' to node 'G2', while i_3 flows up from node 'G2' to node 'c'. Thus, the net resultant current flowing from node 'G2' to node 'c' is $i_3 - i_2$ which must equal the given current 7A,

$$i_3 - i_2 = 7.$$

We emphasize that the presence of current sources (either independent or dependent) always inter-relate the mesh currents, and thus simplify the analysis.

Since there exists one relationship among the mesh currents owing to the presence of one current source, we need to write only two KVL mesh equations. One simple KVL mesh equation can be written easily for the mesh abG1Ga,

$$-20i_1 - 60(i_1 - i_2) = 0.$$

Since the voltage across the current source is unknown, neither of the two mesh equations (one for bcG2G1b and the other for cdG3G2c) can readily be written without introducing other unknown variables. However, the so called **super mesh** equation for bcdG3G2G1b (combination of two meshes bcG2G1b and cdG3G2c) can be readily written as

$$-15i_2 - 5i_3 - 60(i_2 - i_1) = 0.$$

The solution of the above equations yields $i_1 = -0.75$ A, $i_2 = -1$ A, and $i_3 = 6$ A. Once i_1, i_2, and i_3 are known, all the branch currents can be easily computed and are as given in the circuit of Figure 2b in the previous page.

A combination of two or more meshes gives a loop or a closed path which is called a Super mesh. The name Super mesh is a misnomer, because by definition it is not a mesh as it encloses other meshes.

Example 3 by Super position Method: Solve the circuit shown below in Figure 3a. We can solve the circuit by Super position Method. We can first set the current source value to zero, that is open it. This yields the circuit in Example 1. We can solve the circuit in Example 1 for all the currents. Next, we can set the voltage source value to zero, that is short it. This yields the circuit in Example 2. We can solve this circuit as well for all the currents. Then, the total solution is the algebraic combination (with signs taken into account) of the solutions in Examples 1 and 2. This resultant solution is given in the circuit of Figure 3b.

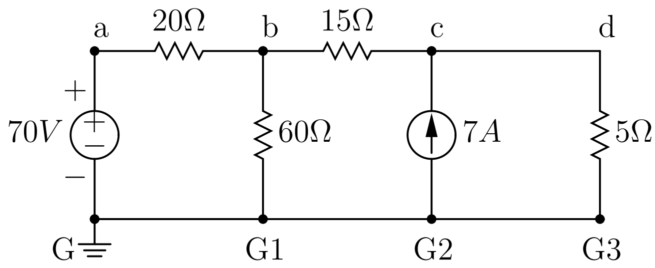


Figure 3a

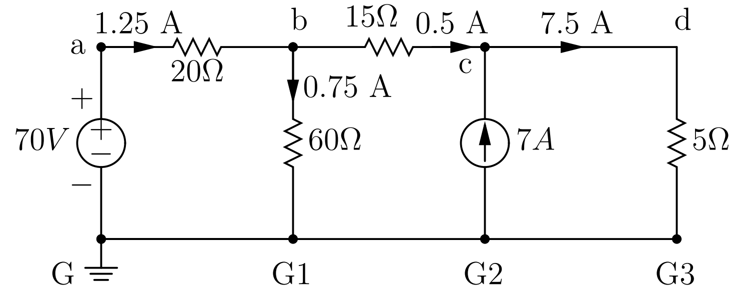
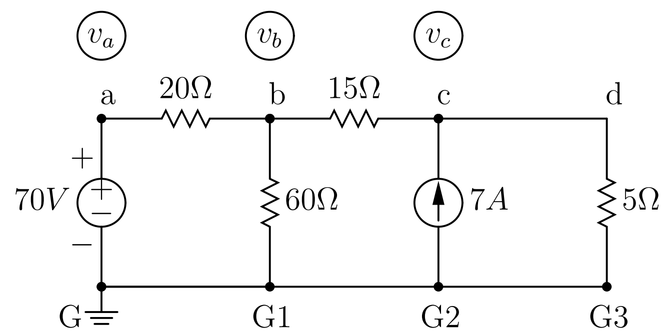


Figure 3b

Example 3 by Node Voltage Method: In order to solve the circuit shown by **node voltage method**, the voltage of each node with respect to the reference G is marked in a circle as shown. Note that the voltage source 70 V readily tells us that $v_a = 70$ V.



We emphasize that the presence of voltage sources (either independent or dependent) always inter-relate the node voltages, and thus simplify the analysis.

By summing all the currents leaving the node 'b' and equating the sum to zero, we get

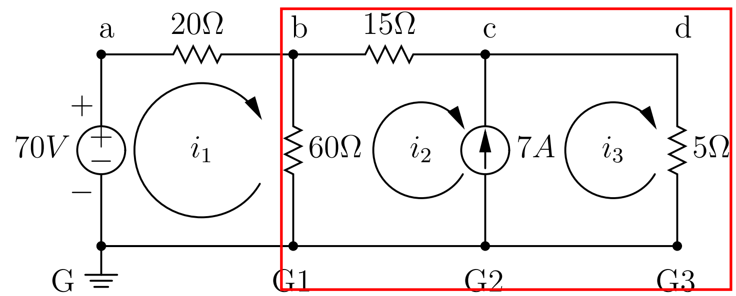
$$\frac{v_b - 70}{20} + \frac{v_b - 0}{60} + \frac{v_b - v_c}{15} = 0.$$

Similarly, the KCL equation at node 'c' can be written as

$$\frac{v_c - v_b}{15} + \frac{v_c - 0}{5} - 7 = 0.$$

The solution of the above equations yields $v_b = 45$ V and $v_c = 37.5$ V. Once v_b and v_c are known, all the branch currents can be easily computed and are given above in the circuit of Figure 3b.

Example 3 by Mesh Current Method: The circuit of Figure 3a has three meshes, and hence has three circulating currents i_1, i_2, and i_3 as marked. The direction of mesh currents is chosen arbitrarily.



We can relate the current source to the chosen mesh currents. As in Example 2, we note that there are two mesh currents flowing through the current source, i_2 flows down from node 'c' to node 'G2', while i_3 flows up from node 'G2' to node 'c'. Thus, the net resultant current flowing from node 'G2' to node 'c' is $i_3 - i_2$ which must equal the given current 7A,

$$i_3 - i_2 = 7.$$

As in Example 2, we emphasize that the presence of current sources (either independent or dependent) always inter-relate the mesh currents, and thus simplify the analysis.

Since there exists one relationship among the mesh currents owing to the presence of one current source, we need to write only two KVL mesh equations. As in Example 2, one simple KVL mesh equation can be written easily for the mesh abG1Ga,

$$70 - 20i_1 - 60(i_1 - i_2) = 0.$$

Once again, as in Example 2, since the voltage across the current source is unknown, neither of the two mesh equations (one for bcG2G1b and the other for cdG3G2c) can readily be written without introducing other unknown variables. However, the so called **super mesh bcdG3G2G1b (combination of two meshes bcG2G1b and cdG3G2c)** can be readily written as

$$-15i_2 - 5i_3 - 60(i_2 - i_1) = 0.$$

The solution of the above equations yields $i_1 = 1.25$ A, $i_2 = 0.5$ A, and $i_3 = 7.5$ A. Once i_1, i_2, and i_3 are known, all the branch currents can be easily computed and are as given in the circuit of Figure 3b in the previous page.

Example 4: Solve the circuit shown on the right in Figure 4a.

In this circuit, there is one independent source and one dependent source. In an attempt to solve this circuit by super position method, if we set the independent voltage source to zero, only the dependent source remains. Any circuit with no independent sources is a dead circuit. All the variables would have zero values. Thus the super position method cannot be applied to this circuit. One has to solve this circuit by other means.

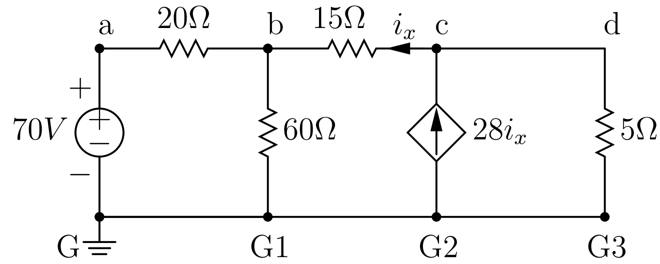
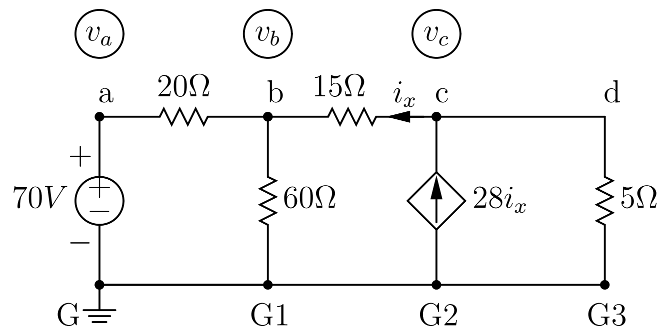


Figure 4a

Example 4 by Node Voltage Method: In order to solve the circuit shown by **node voltage method**, the voltage of each node with respect to the reference G is marked in a circle as shown. As in Example 3, the voltage source 70 V readily tells us that $v_a = 70$ V. We need to write two KCL node equations to inter-relate v_b and v_c.



Before we proceed to write the two KCL node equations, let us first see how the controlling current i_x is related to the node voltages v_b and v_c. Clearly,

$$i_x = \frac{v_c - v_b}{15}.$$

Thus, the dependent current source $28i_x$ is given by

$$28i_x = 28\frac{v_c - v_b}{15}.$$

Next, by summing all the currents leaving the node 'b' and equating the sum to zero, we get

$$\frac{v_b - 70}{20} + \frac{v_b - 0}{60} + \frac{v_b - v_c}{15} = 0.$$

Similarly, the KCL equation at node 'c' can be written as

$$\frac{v_c - v_b}{15} + \frac{v_c - 0}{5} - 28i_x = 0 \quad \Rightarrow \quad \frac{v_c - v_b}{15} + \frac{v_c - 0}{5} - 28\frac{v_c - v_b}{15} = 0.$$

The solution of the above equations yields $v_b = 60$ V and $v_c = 67.5$ V. Once v_b and v_c are known, all the branch currents can be easily computed and are given as follows:

The current of dependent source is $28i_x = 28\frac{v_c - v_b}{15} = 28\frac{67.5-60}{15} = 14$ A.

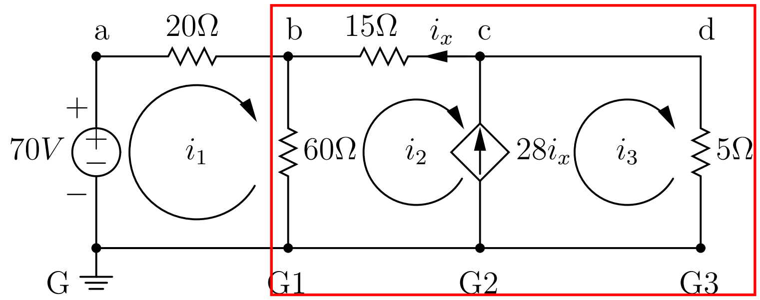
The current in 20Ω resistance leaving node 'a' towards node 'b' is $\frac{v_a - v_b}{20} = \frac{70-60}{20} = 0.5$ A.

The current in 60Ω resistance leaving node 'b' towards node 'G1' is $\frac{v_b - 0}{60} = \frac{60-0}{60} = 1$ A.

The current in 15Ω resistance leaving node 'c' towards node 'b' is $\frac{v_c - v_b}{15} = \frac{67.5-60}{15} = 0.5$ A.

The current in 5Ω resistance leaving node 'c' towards node 'G3' is $\frac{v_c - 0}{5} = \frac{67.5-0}{5} = 13.5$ A.

Example 4 by Mesh Current Method: The circuit of Figure 4a has three meshes, and hence has three circulating currents i_1, i_2, and i_3 as marked. The direction of mesh currents is chosen arbitrarily.



We can relate the current source to the chosen mesh currents. As in Examples 2 and 3, we note that there are two mesh currents flowing through the dependent current source, i_2 flows down from node 'c' to node 'G2', while i_3 flows up from node 'G2' to node 'c'. Thus, the net resultant current flowing from node 'G2' to node 'c' is $i_3 - i_2$ which must equal the given current current $28i_x$. This calls for the computation of the controlling current i_x in terms of the mesh currents. Clearly,

$$i_x = -i_2.$$

This means that

$$i_3 - i_2 = 28i_x = -28i_2 \quad \Rightarrow \quad i_3 + 27i_2 = 0.$$

As in previous examples, we emphasize that the presence of current sources (either independent or dependent) always inter-relate the mesh currents, and thus simplify the analysis.

We have one relationship inter-relating the mesh currents. We need to develop two more relationships among the mesh currents. This can be done by writing two KVL mesh equations. As in Examples 2 and 3, one simple KVL mesh equation can be written easily for the mesh abG1Ga,

$$70 - 20i_1 - 60(i_1 - i_2) = 0.$$

Once again, as in Examples 2 and 3, since the voltage across the current source is unknown, neither of the two mesh equations (one for bcG2G1b and the other for cdG3G2c) can readily be written without introducing other unknown variables. However, the so called **super mesh bcdG3G2G1b (combination of two meshes bcG2G1b and cdG3G2c)** can be readily written as

$$-15i_2 - 5i_3 - 60(i_2 - i_1) = 0.$$

The solution of the above equations yields $i_1 = 0.5$ A, $i_2 = -0.5$ A, and $i_3 = 13.5$ A. Once i_1, i_2, and i_3 are known, all the branch currents can be easily computed.

Example 5: Solve the circuit shown on the right in Figure 5a. As in Example 4, super position method cannot be applied to this circuit. One has to solve this circuit by other means.

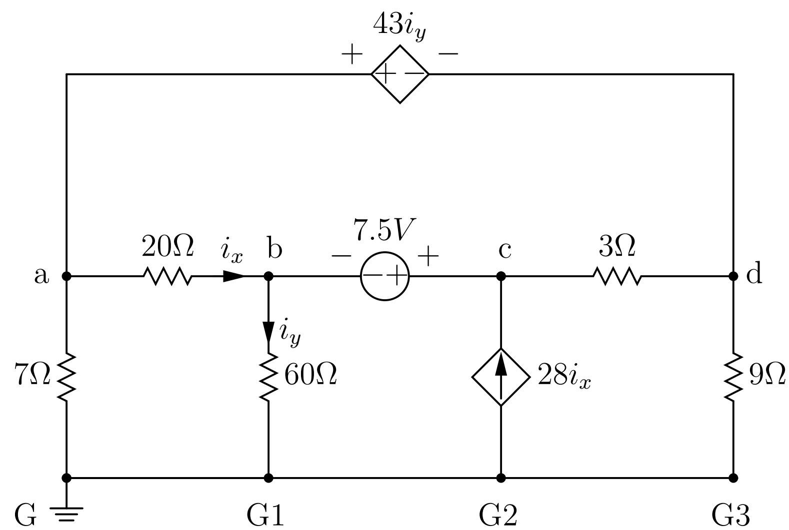


Figure 5a

The solution of the circuit of Figure 5a turns out to be as given in the circuit of Figure 5b.

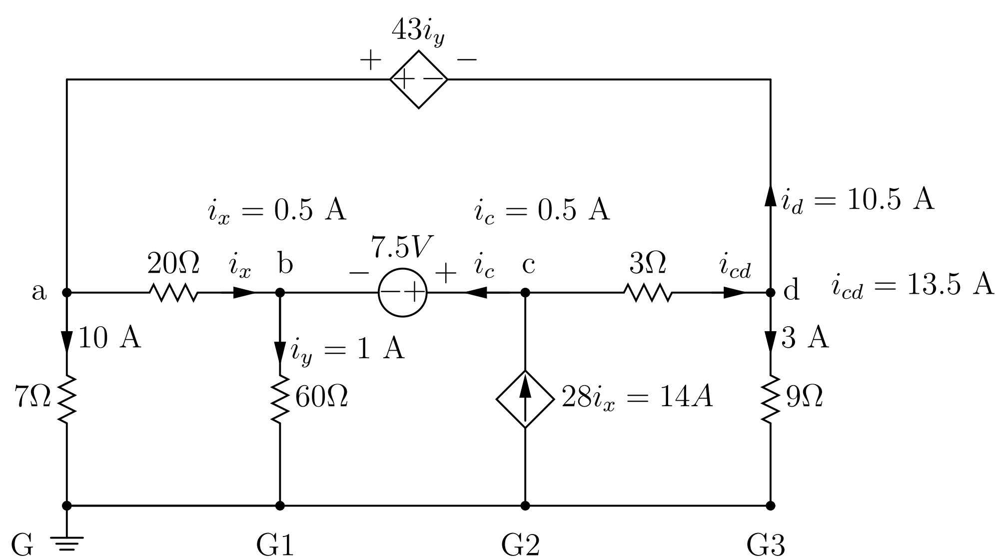
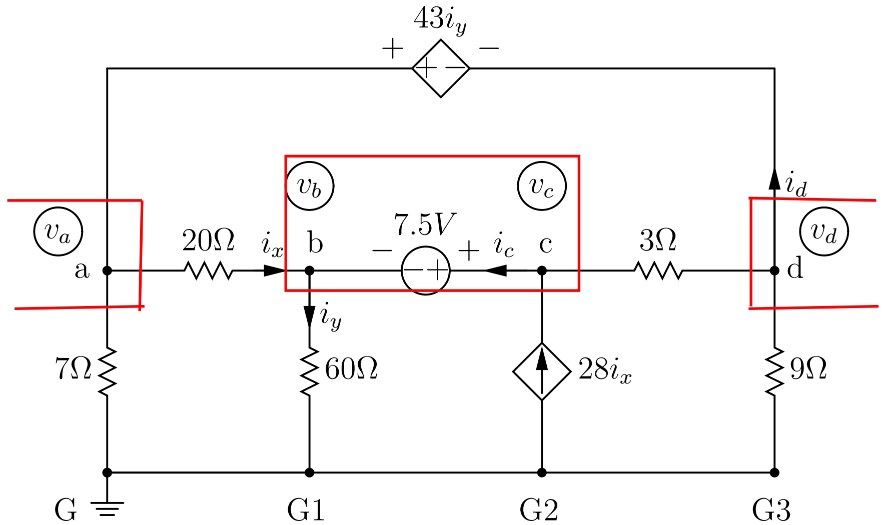


Figure 5b

Example 5 by Node Voltage Method: In order to solve the circuit shown by **node voltage method**, the voltage of each node with respect to the reference G is marked in a circle as shown. Some of these node voltages can be deciphered simply by looking at the given voltage sources. For others, we need to write appropriate KCL node equations.



We see easily that $v_b + 7.5 = v_c$ and $v_a - 43i_y = v_d$.
In order to rewrite the second of the above two equations only in terms of the basic node voltages, we need to write i_y in terms of the node voltages. Clearly,

$$i_y = \frac{v_b - 0}{60} = \frac{v_b}{60}.$$

Thus,

$$v_d = v_a - 43i_y = v_a - 43\frac{v_b}{60}.$$

By now, we developed two inter-relationships among the node voltages v_a, v_b, v_c and v_d by just looking at the two voltage sources (both independent and dependent). We emphasize that the presence of voltage sources (either independent or dependent) always inter-relate the node voltage variables, and in this process simplifies the analysis.

We need to write next two more relationships to solve for v_a, v_b, v_c and v_d. Before we do so, let us relate the dependent current source $28i_x$ in terms of the basic node voltages. This is needed to relate all the unknown variables to the basic node voltage variables. In this connection, we observe that

$$i_x = \frac{v_a - v_b}{20} \quad \Rightarrow \quad 28i_x = 28\frac{v_a - v_b}{20}.$$

Now we proceed to obtain two more relationships among the basic node voltage variables. We do this by writing KCL node equations. However, **when we attempt to write a KCL equation at any single node, we see a voltage source connected to that node and the current in it is unknown. So we need to resort to what is called a super node equation which is a combination of two or mode KCL node equations**. In this regard, at first we can write the KCL node equation at node ‘d’ to get

$$\frac{v_d - v_c}{3} + \frac{v_d - 0}{9} + i_d = 0,$$

where i_d is the current leaving the node ‘d’ towards node ‘a’ via the dependent voltage source $43i_y$. This calls for writing another node equation to eliminate i_d. We can write the KCL node equation at node ‘a’ to get,

$$\frac{v_a - 0}{7} + \frac{v_a - v_b}{20} - i_d = 0.$$

By adding the above two node equations, we get what can be called a super node equation,

$$\frac{v_d - v_c}{3} + \frac{v_d - 0}{9} + \frac{v_a - 0}{7} + \frac{v_a - v_b}{20} = 0. \text{ (Equation at both nodes 'a' and 'd')}$$

By now, we developed three inter-relationships among the four basic node voltage variables. We need to write one more equation. Let us write the KCL node equation at node 'c',

$$\frac{v_c - v_d}{3} - 28i_x + i_c = 0 \quad \Rightarrow \quad \frac{v_c - v_d}{3} - 28\frac{v_a - v_b}{20} + i_c = 0,$$

where i_c is the current leaving the node 'c' towards node 'b' via the voltage source 7.5 V. This calls for writing another node equation to eliminate i_c. By writing another KCL node equation, this time at node 'b', we get

$$\frac{v_b - v_a}{20} + \frac{v_b - 0}{60} - i_c = 0.$$

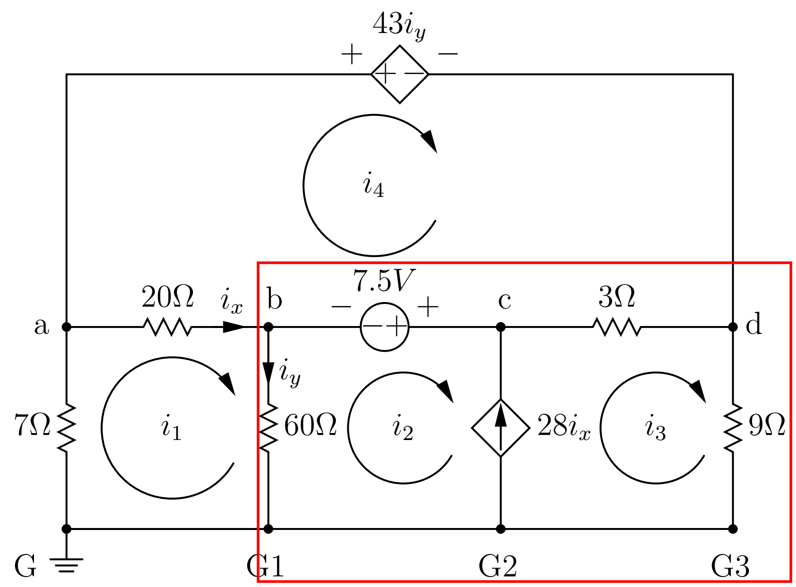
By adding the above two node equations, we get what can be called another super node equation,

$$\frac{v_b - v_a}{20} + \frac{v_b - 0}{60} + \frac{v_c - v_d}{3} - 28\frac{v_a - v_b}{20} = 0. \text{ (Equation at both nodes 'b' and 'c')}$$

With this, we have written four equations in four unknowns v_a, v_b, v_c and v_d. By solving these four equations, we obtain $v_a = 70$ V, $v_b = 60$ V, $v_c = 67.5$ V and $v_d = 27$ V. Once we know these values, we can solve for all branch currents as given in the circuit of Figure 5b in previous page.

A combination of two or more node equationss gives a Super node equation. The name Super node is a misnomer, because by definition it is not a simple node.

Example 5 by Mesh Current Method: The circuit of Figure 5a has four meshes, and hence has four circulating currents i_1, i_2, i_3, and i_4 as marked. The direction of mesh currents is chosen arbitrarily.



There is one dependent current source. In this current source, i_2 flows down from node 'c' to node 'G2', while i_3 flows up from node 'G2' to node 'c'. Thus, the net resultant current flowing from node 'G2' to node 'c' is $i_3 - i_2$ which must equal the given current current $28i_x$. This calls for relating the controlling current i_x to the basic mesh currents. This is needed to relate all the unknown variables to the basic mesh current variables. Clearly,

$$i_x = i_1 - i_4.$$

This means that

$$i_3 - i_2 = 28i_x = 28(i_1 - i_4) \quad \Rightarrow \quad 28i_1 + i_2 - i_3 - 28i_4 = 0.$$

Thus we have one relationship inter-relating the mesh currents.
We emphasize that the presence of current sources (either independent or dependent) always inter-relate the mesh current variables, and in this process simplifies the analysis.
We need to develop three more relationships among the mesh currents. We can write easily two KVL mesh equations, one for the mesh abG1Ga, and another for the mesh adcba,

$$-20(i_1 - i_4) - 60(i_1 - i_2) - 7i_1 = 0,$$

and

$$-43i_y - 3(i_4 - i_3) - 7.5 - 20(i_4 - i_1) = 0.$$

We note that $i_y = i_1 - i_2$. Substituting for i_y in the previous equation, we get

$$-43(i_1 - i_2) - 3(i_4 - i_3) - 7.5 - 20(i_4 - i_1) = 0 \quad \Rightarrow \quad -23i_1 + 43i_2 + 3i_3 - 23i_4 - 7.5 = 0.$$

We need to write one more equation. Once again, as in Examples 2, 3 and 4, since the voltage across the current source is unknown, neither of the two mesh equations (one for bcG2G1b and the other for cdG3G2c) can readily be written without introducing other unknown variables. However, the so called super mesh bcdG3G2G1b (combination of two meshes bcG2G1b and cdG3G2c) can be readily written as

$$7.5 - 3(i_3 - i_4) - 9i_3 - 60(i_2 - i_1) = 0.$$

We have written four relationships among the mesh currents. The solution of these equations yields $i_1 = -10$ A, $i_2 = -11$ A, $i_3 = 3$ A, and $i_4 = -10.5$ A. Once i_1, i_2, i_3 and i_4 are known, all the branch currents can be computed easily.

Example 6: Solve the circuit shown on the right in Figure 6a. As in Examples 4 and 5, super position method cannot be applied to this circuit. One has to solve this circuit by other means.

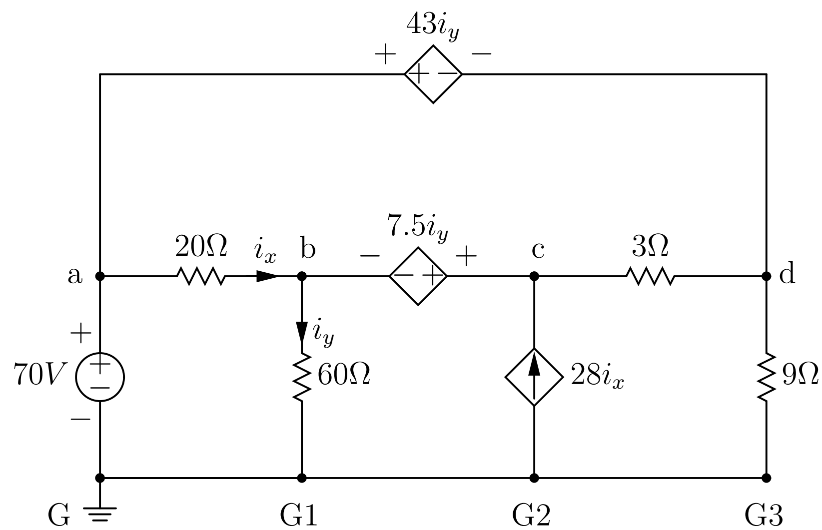


Figure 6a

The solution of the circuit of Figure 6a turns out to be as given in the circuit of Figure 6b.

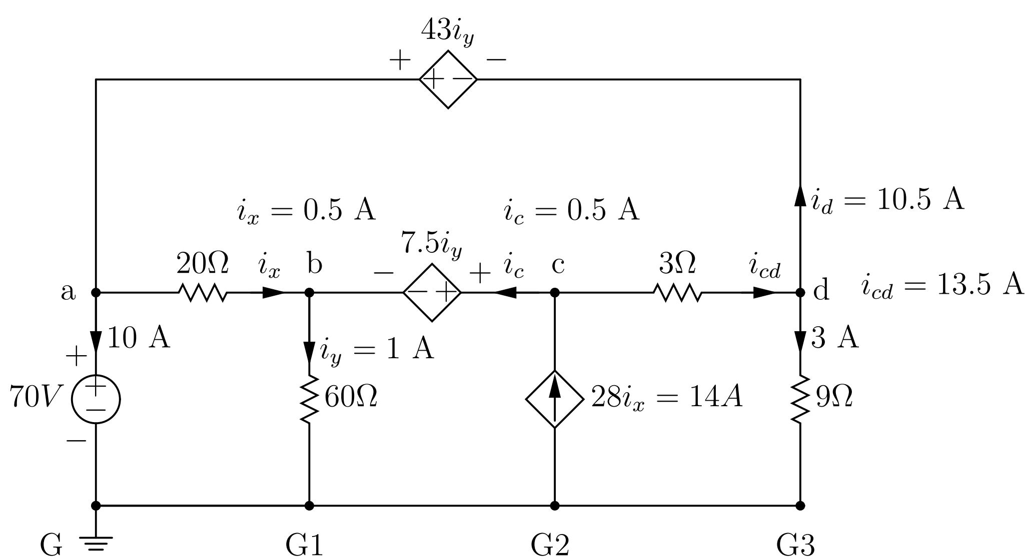
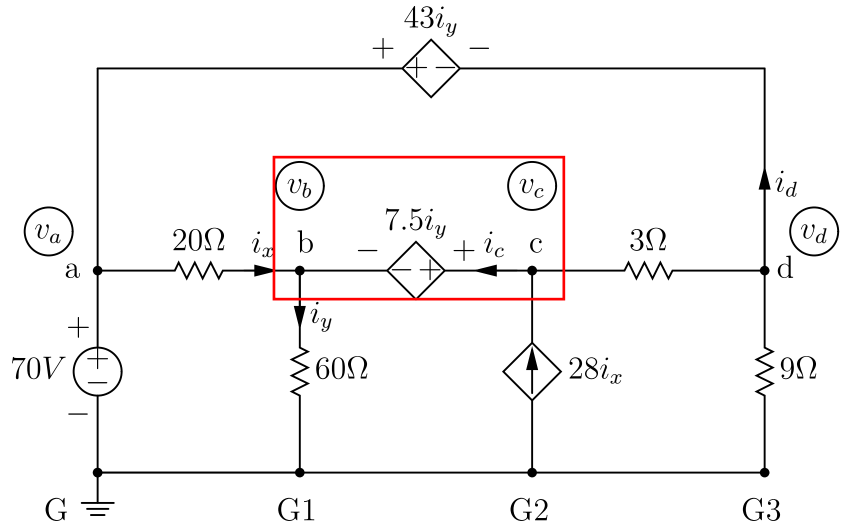


Figure 6b

Example 6 by Node Voltage Method: In order to solve the circuit shown by **node voltage method**, the voltage of each node with respect to the reference G is marked in a circle as shown. Some of these node voltages can be deciphered simply by looking at the given voltage sources. For others, we need to write appropriate KCL node equations.



We see easily that $v_a = 70V$.
We see also that v_b and v_c are related by the dependent voltage source $7.5i_y$, namely

$$v_c = v_b + 7.5i_y.$$

In order to rewrite the above equation only in terms of the basic node voltages, we need to write i_y in terms of the node voltages. Clearly,

$$i_y = \frac{v_b - 0}{60}.$$

Thus,

$$v_c = v_b + 7.5i_y = v_b + 7.5\frac{v_b}{60}.$$

Next, we observe that v_a and v_d are related by the dependent voltage source $43i_y$, namely

$$v_d = v_a - 43i_y \quad \Rightarrow \quad v_d = v_a - 43\frac{v_b}{60}.$$

By now, we developed three inter-relationships among the node voltages v_a, v_b, v_c and v_d by just looking at the three voltage sources (both independent and dependent) in the given circuit. We need to write one more relationship to solve for them. Before we do so, let us determine the dependent current source $28i_x$ in terms of the basic node voltages. In this connection, we observe that

$$i_x = \frac{v_a - v_b}{20} = \frac{70 - v_b}{20} \quad \Rightarrow \quad 28i_x = 28\frac{70 - v_b}{20}.$$

Now we proceed to write a KCL node equation. However, when we attempt to write such an equation at any single node, we see a voltage source connected to that node and the current in it is unknown. So we need to resolve to what is called a super node equation which is a combination of two or mode KCL node equations. In this regard, at first we can write the KCL node equation at node 'c', we get

$$\frac{v_c - v_d}{3} - 28i_x + i_c = 0 \quad \Rightarrow \quad \frac{v_c - v_d}{3} - 28\frac{70 - v_b}{20} + i_c = 0,$$

where i_c is the current leaving the node 'c' towards node 'b' via the dependent voltage source $7.5i_y$. This calls for writing another equation for i_c. By writing another KCL node equation, this time at node 'b', we get

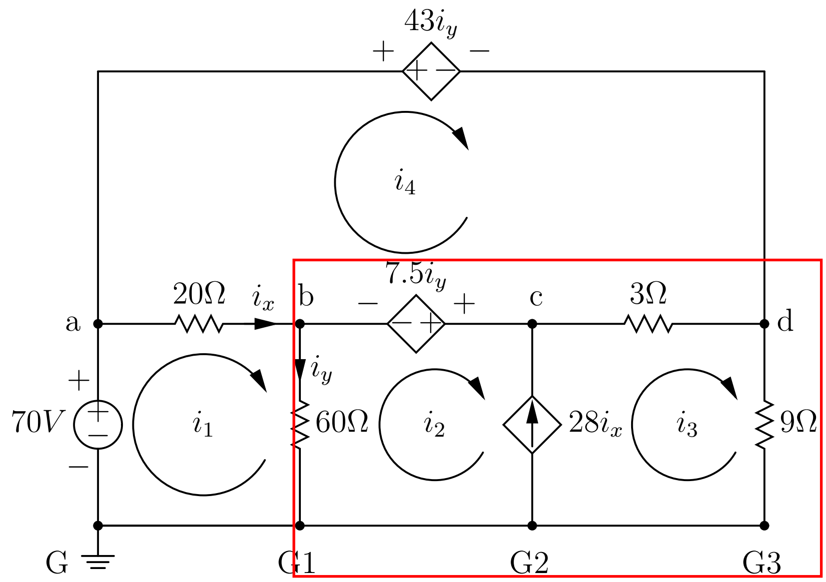
$$\frac{v_b - v_a}{20} + \frac{v_b - 0}{60} - i_c = 0.$$

By adding the above two node equations, we get what can be called a super node equation,

$$\frac{v_b - v_a}{20} + \frac{v_b - 0}{60} + \frac{v_c - v_d}{3} - 28\frac{70 - v_b}{20} = 0.$$

With this, we have written four equations in four unknowns v_a, v_b, v_c and v_d. By solving these four equations, we obtain $v_a = 70$ V, $v_b = 60$ V, $v_c = 67.5$ V and $v_d = 27$ V. Once we know these values, we can solve for all branch currents as given in the circuit of Figure 6b in previous page.

Example 6 by Mesh Current Method: The circuit of Figure 6a has four meshes, and hence has four circulating currents i_1, i_2, i_3, and i_4 as marked. The direction of mesh currents is chosen arbitrarily.



As in previous examples, we note that there are two mesh currents flowing through the dependent current source, i_2 flows down from node 'c' to node 'G2', while i_3 flows up from node 'G2' to node 'c'. Thus, the net resultant current flowing from node 'G2' to node 'c' is $i_3 - i_2$ which must equal the given current current $28i_x$. This calls for the computation of the controlling current i_x as it is related to the mesh currents. Clearly,

$$i_x = i_1 - i_4.$$

This means that

$$i_3 - i_2 = 28i_x = 28(i_1 - i_4) \quad \Rightarrow \quad 28i_1 + i_2 - i_3 - 28i_4 = 0.$$

Thus we have one relationship inter-relating the mesh currents. We need to develop three more relationships among the mesh currents. We can write easily two KVL mesh equations, one for the mesh abG1Ga, and another for the mesh adcba,

$$-20(i_1 - i_4) - 60(i_1 - i_2) + 70 = 0,$$

and

$$-43i_y - 3(i_4 - i_3) - 7.5i_y - 20(i_4 - i_1) = 0.$$

We note that $i_y = i_1 - i_2$. Substituting for i_y in the previous equation, we get

$$-50.5(i_1 - i_2) - 3(i_4 - i_3) - 20(i_4 - i_1) = 0.$$

We need to write one more equation. Once again, as in Examples 2, 3, 4, and 5, since the voltage across the dependent current source is unknown, neither of the two meshes (bcG2G1b and cdG3G2c) can readily be written. However, the so called super mesh bcdG3G2G1b (combination of two meshes bcG2G1b and cdG3G2c) can be readily written as

$$7.5i_y - 3(i_3 - i_4) - 9i_3 - 60(i_2 - i_1) = 0 \quad \Rightarrow \quad 7.5(i_1 - i_2) - 3(i_3 - i_4) - 9i_3 - 60(i_2 - i_1) = 0.$$

The solution of the above equations yields $i_1 = -10$ A, $i_2 = -11$ A, $i_3 = 3$ A, and $i_4 = -10.5$ A. Once i_1, i_2, i_3 and i_4 are known, all the branch currents can be computed easily.

We can now summarize the philosophy of Node Voltage Method and Mesh Current Method as given in next page.

Let B be the number of branches and N be the number of nodes.

Node Voltage Method

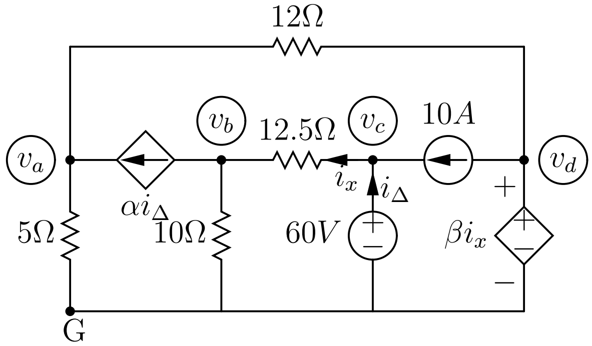
1. Select a reference node. Each of other nodes has a voltage with respect to the reference node. Assign $N-1$ number of node voltages, one to each of the nodes other than the reference node. These $N-1$ node voltage variables are the basic variables we like to determine.

2. Determine all the controlling variables in terms of basic variables.

3. Let us suppose there are X number of independent or dependent voltage sources exist in the circuit. These voltage sources yield X relationships between the basic $N-1$ node voltage variables.

4. The number of additional equations that need to be written equals the number of basic variables minus the number of inter-relationships among the basic variables determined in the above step. That is, write down $N-1-X$ number of independent KCL node equations by examining each node.

5. Solve all the above $N-1$ simultaneous equations to determine all the $N-1$ basic node voltage variables.



Mesh Current Method

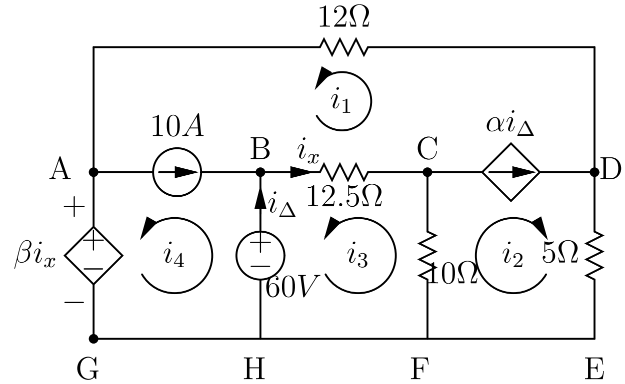
1. For a planar circuit, there always exist $B-(N-1)$ number of meshes or windows. Assign a mesh current variable that circulates around the frame of each mesh, the direction of the circulating current is selected arbitrarily as desired. Since there are $B-(N-1)$ number of meshes, there are $B-(N-1)$ number of basic mesh current variables, and these are the basic variables we like to determine.

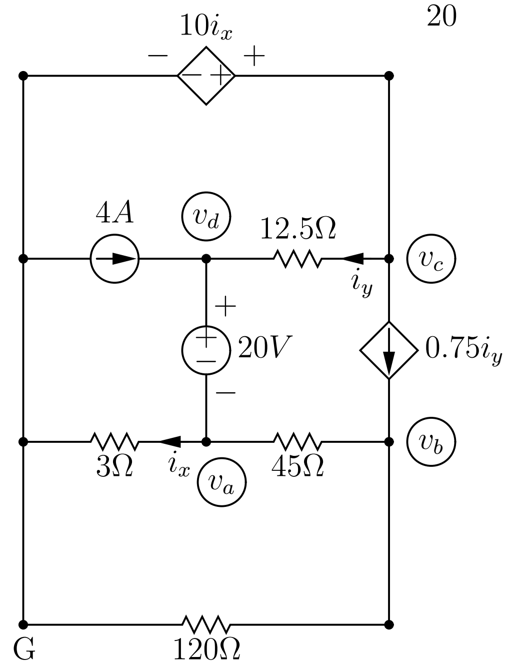
2. Determine all the controlling variables in terms of basic variables.

3. Let us suppose there are X number of independent or dependent current sources exist in the circuit. These current sources yield X relationships between the basic $B-(N-1)$ basic mesh current variables.

4. The number of additional equations that need to be written equals the number of basic variables minus the number of inter-relationships among the basic variables determined in the above step. That is, write down $B-(N-1)-X$ number of independent KVL equations by examining each mesh.

5. Solve all the above $B-(N-1)$ simultaneous equations to determine all the $B-(N-1)$ basic current variables.





Example 7 by Node Voltage Method: We would like to use nodal analysis to solve the circuit shown on the right. Using G as the ground or reference, the node voltages v_a, v_b, v_c, and v_d are marked in circles near the nodes. To start with, all these node voltages can be considered as unknown although values of some of them can be obtained easily. Note that we have four unknowns and hence we need to write four equations in addition to the equations that define the controlling variables. The following steps illustrate a systematic way of writing the necessary equations to determine all the node voltages.

1. There are controlling variables in the circuit. Write one equation for each one of them in terms of basic node voltages v_a, v_b, v_c, and v_d.

$$i_x = \frac{v_a}{3} \text{ and } i_y = \frac{v_c - v_d}{12.5}.$$

2. There are voltage sources in the circuit (both independent and dependent). These voltage sources yield relationships among the basic node voltage variables. Write these relationships below. The dependent sources are controlled by certain variables, substitute for them using the results of Step 1 and re-write the equations of this step so that they depend only on node voltages v_a, v_b, v_c, and v_d.

$$v_d - v_a = 20V \text{ and } v_c = 10i_x = 10\frac{v_a}{3}.$$

3. The above two steps yield two equations among the basic node voltage variables. In addition to these two equations, we need to write two more equations by examining appropriate nodes.
   Write one KCL equation for the node whose voltage is v_b. Substitute for any controlling variables using the results of Step 1 and re-write the equation.

$$\frac{v_b}{120} + \frac{v_b - v_a}{45} = 0.75i_y = 0.75\frac{v_c - v_d}{12.5}.$$

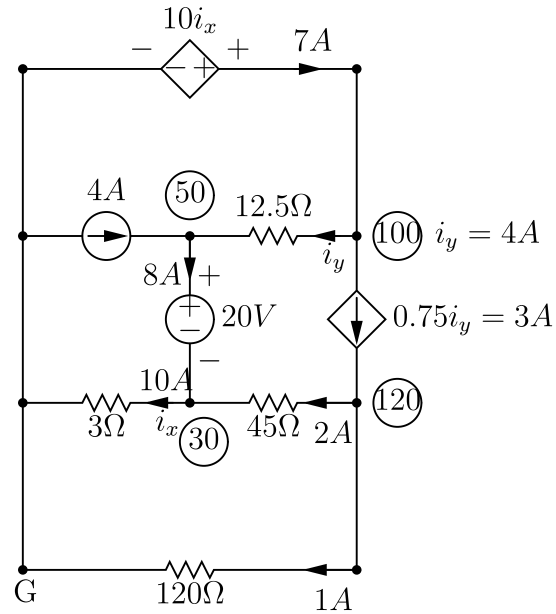
   Write another KCL equation at another node or multiple of nodes (super node equation). Specify which node equation(s) you wrote.

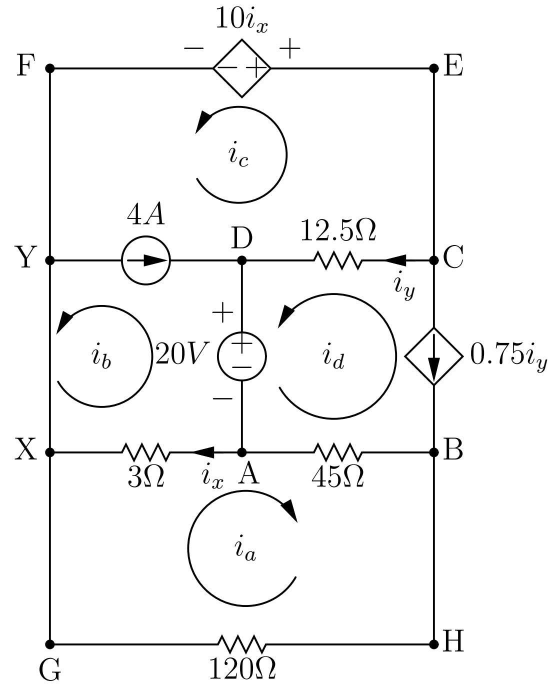
$$\frac{v_a}{3} + \frac{v_a - v_b}{45} - 4 + \frac{v_d - v_c}{12.5} = 0.$$

By solving all the above equations, we get

$$v_a = 30V, \quad v_b = 120V, \quad v_c = 100V, \ \text{ and } \ v_d = 50V, \quad \text{while} \quad i_x = 10A \ \text{ and } \ i_y = 4A.$$

Once all the node voltages are known, it is easy to compute all the branch currents and branch voltages as shown in the circuit given below.





Example 7 by Mesh Current Method: We would like to use mesh analysis to solve the circuit shown on the right. The mesh currents i_a, i_b, i_c, and i_d are marked as shown. To start with, all these mesh currents can be considered as unknown although values of some of them can be obtained easily. Note that we have four unknowns and hence we need to write four equations in addition to the equations that define the controlling variables. The following steps illustrate a systematic way of writing the necessary equations to determine all the mesh currents:

1. There are controlling variables in the circuit. Write one equation for each one of them in terms of basic mesh currents i_a, i_b, i_c, and i_d.

$$i_x = -(i_a + i_b) \quad \text{and} \quad i_y = i_d - i_c.$$

2. There are current sources in the circuit (both independent and dependent). These current sources yield relationships among the basic mesh current variables. Write these relationships below. The dependent sources are controlled by certain variables, substitute for them using the results of Step 1 and re-write the equations of this step so that they depend only on mesh currents i_a, i_b, i_c, and i_d.

$$4 = i_c - i_b \quad \text{and} \quad 0.75i_y = 0.75(i_d - i_c) = -i_d.$$

3. The above two steps yield two equations among the basic mesh current variables. In addition to these two equations, we need to write two more equations by examining appropriate meshes.
   Write one equation for the first mesh, i.e., the mesh GXABHG.

$$-3(i_a + i_b) - 45(i_a + i_d) - 120i_a = 0.$$

   Write another mesh equation or multiple of mesh equations (super mesh equation). Specify which mesh equation(s) you wrote.

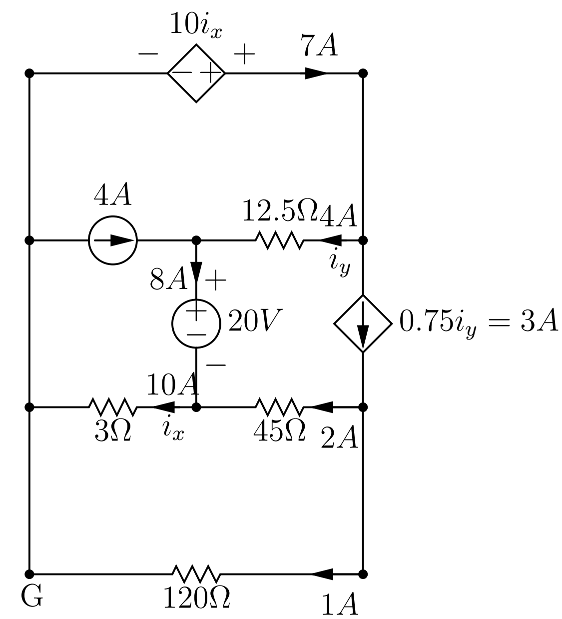
   We write the super mesh XADCEFYX equation,

$$-3(i_a + i_b) + 20 - 12.5(i_c - i_d) - 10i_x = 0.$$

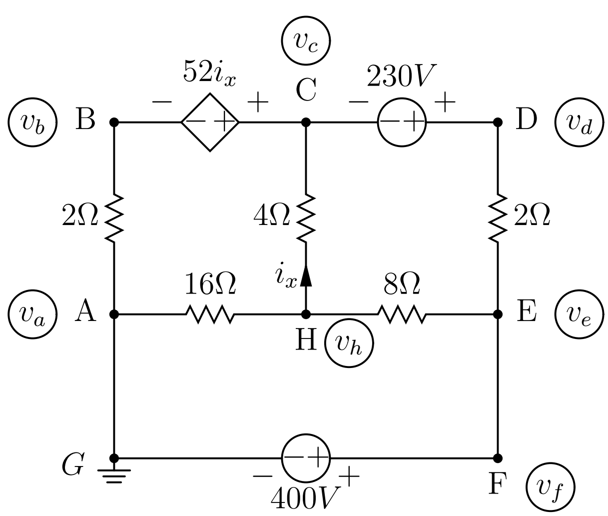
By solving the above equations, we obtain

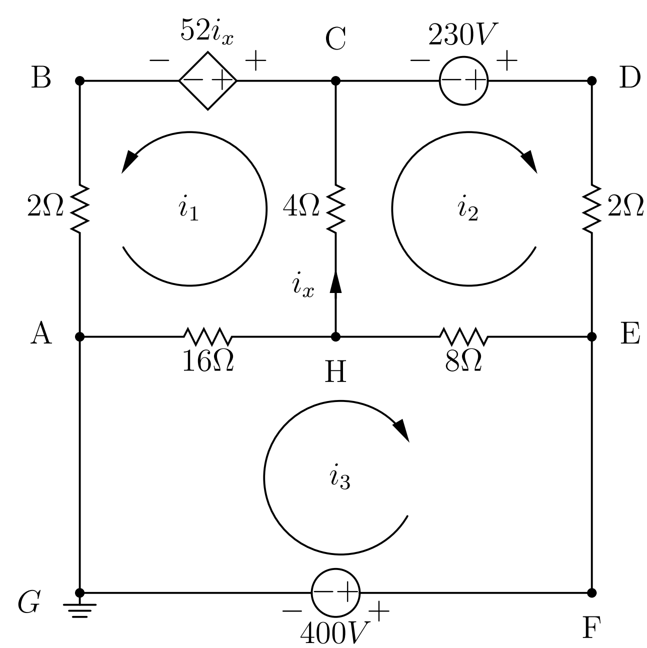
$$i_a = 1\,A,\;\; i_b = -11\,A,\;\; i_c = -7\,A,\;\; \text{and}\;\; i_d = -3\,A, \quad \text{while} \quad i_x = 10\,A, \quad \text{and} \quad i_y = 4\,A.$$

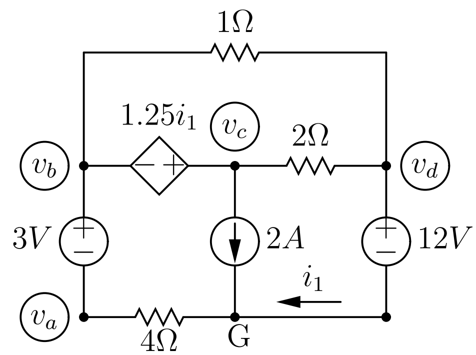
Once i_a, i_b, i_c and i_d are known, all the branch currents can be computed easily as shown below.

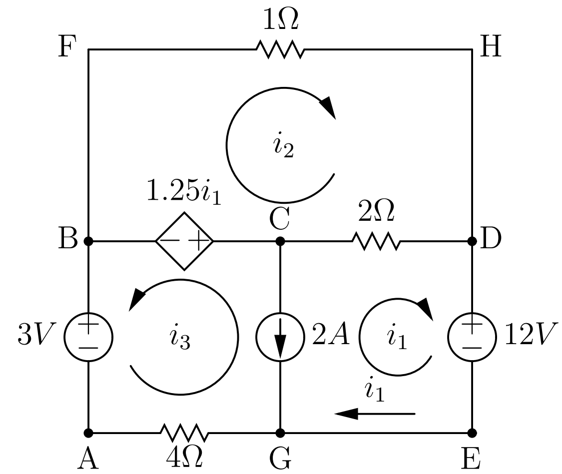


Examples HW, not collected to grade: Write all pertinent equations to solve the circuits shown by either Node Voltage Method or Mesh Current Method as indicated. For all the circuits, the reference node is G.

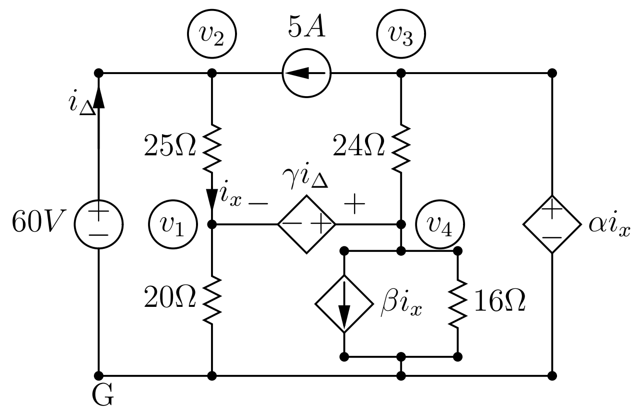


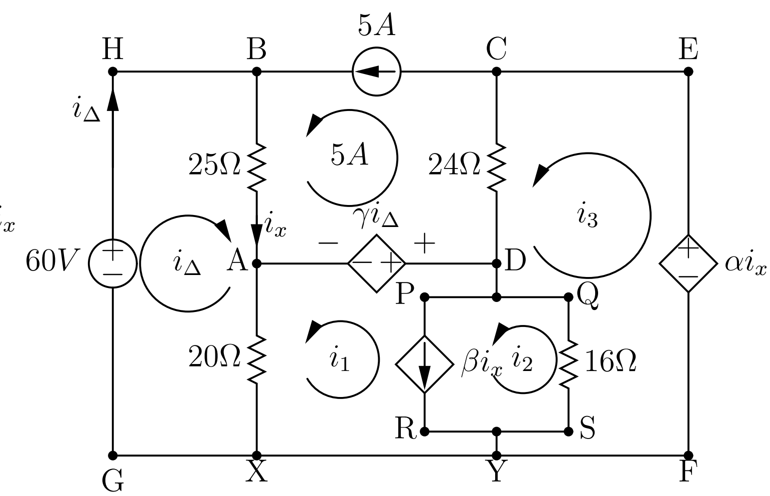






Note: The mesh current i_3 is for the mesh CDQSYFEC.





HW from Nilsson and Riedel 8th and 9th editions

Set up the equations for the following problems. If you can solve them numerically by utilizing your calculator or Matlab, solve them.

Nilsson and Riedel 8th edition:

Nodal Analysis 4.9, 4.10, 4.17, 4.18, 4.19. 4.25, 4.27

Mesh Analysis 4.37, 4.38, 4.43, 4.44, 4.45, 4.46, 4.50

Nilsson and Riedel 9th edition:

Nodal Analysis 4.8, 4.13, 4.18, 4.19, 4.20, 4.25, 4.27

Mesh Analysis 4.38, 4.39, 4.40, 4.45, 4.50, 4.51, 4.56