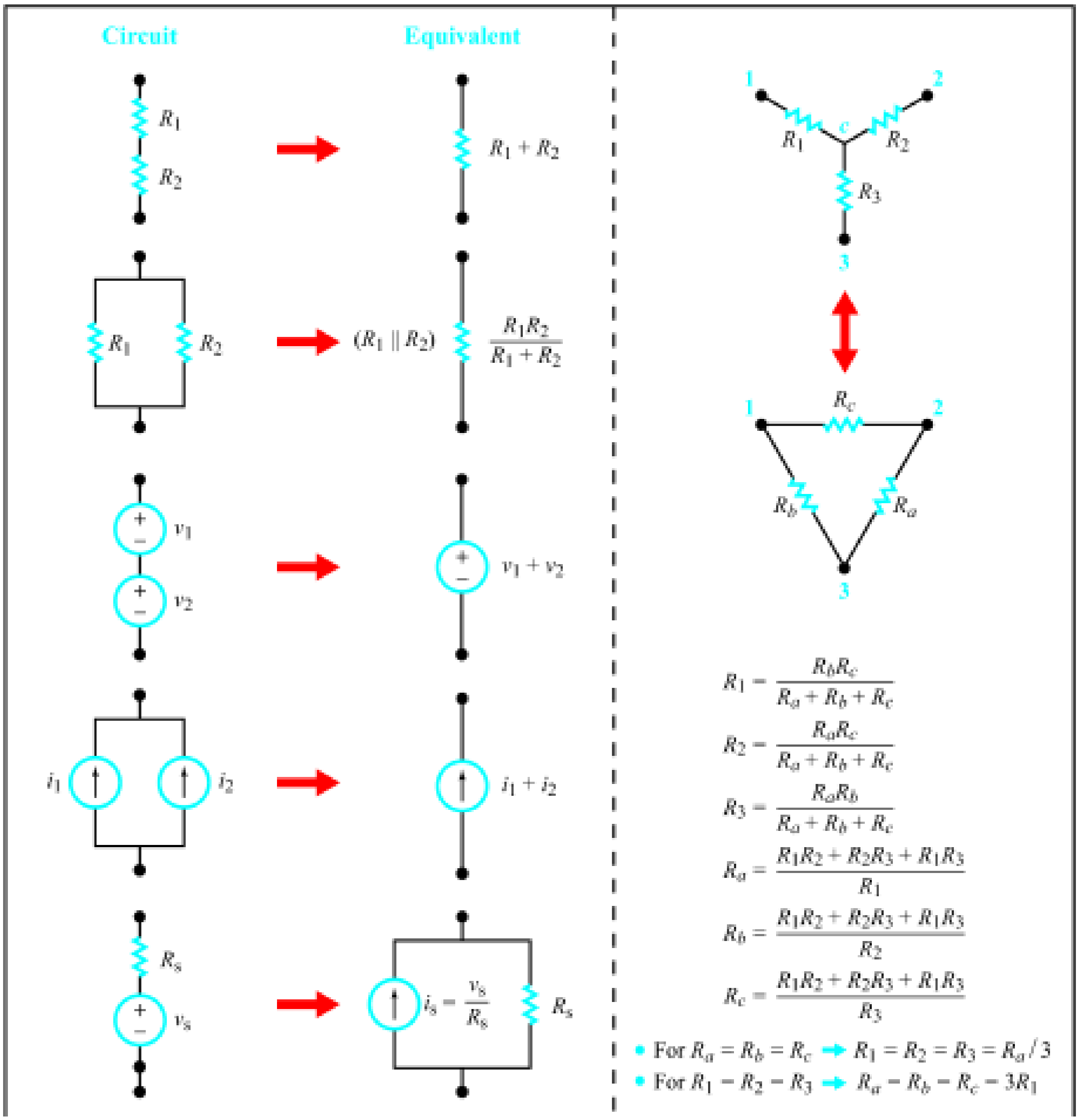
Principles of Electrical Engineering I

Preview of Circuit Equivalents

~~The second part of~~ This Chapter ~~2~~ develops some important circuit equivalents that help to simplify circuit analysis. These include series parallel equivalents, source transformations, and Δ-Y equivalents.



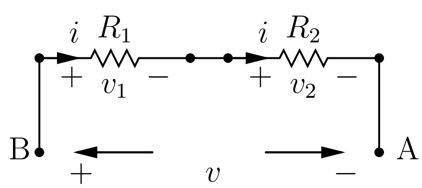
Source Transformations

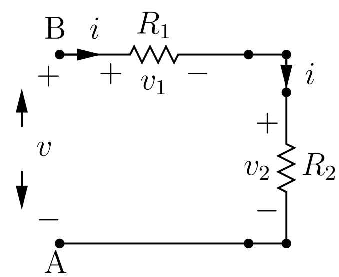
This is done in Chapter 4.

Two resistances interconnected in series: Figure below shows the interconnection of two resistances in series. The physical layout is not important. As seen in the figure, one terminal of the first resistance is connected to one terminal of the second resistance so that the current i flowing in both the resistances is the same. The other two terminals one from each resistance form the external terminals of the connection. One can view both the resistances interconnected together in series as one equivalent resistance. Then, the equivalent resistance between the terminals A and B is given by

$$R_{Eq} = R_1 + R_2. \qquad \text{(Because } v = iR_{Eq} = v_1 + v_2 = i(R_1 + R_2).\text{)}$$

The above equation says that two resistances interconnected in series is equivalent to a single resistance having a value as the sum of two resistances, $\boxed{R_{Eq} = R_1 + R_2}$. Note that the equivalent resistance of two positive resistances in series is greater than either of the two resistances.

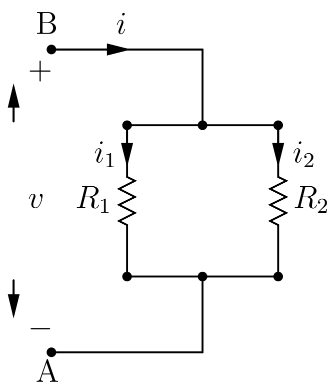


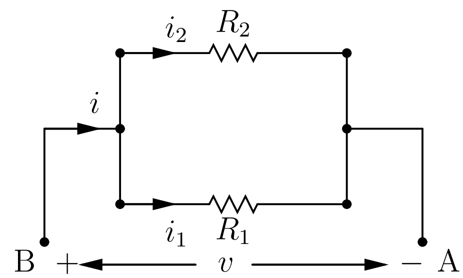


Two resistances interconnected in parallel: Figure below shows the interconnection of two resistances in parallel. Once again the physical layout is not important. As seen in the figure, a pair of two terminals one from each resistance are connected together to form a node or a joint terminal, and similarly another pair of two terminals one from each resistance are again connected together to form another node. Both of these nodes form external terminals. In this case, the voltage v across each resistance is the same, however a current i flowing into a node divides itself into two parts i_1 and i_2. One can view both the resistances interconnected together in parallel as one equivalent resistance. Equivalent resistance of two resistances interconnected in parallel (that is, the resistance between the terminals A and B) is given by

$$\frac{1}{R_{Eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow R_{Eq} = \frac{R_1 R_2}{R_1 + R_2}. \qquad \text{(Because } i = \frac{v}{R_{Eq}} = i_1 + i_2 = \frac{v}{R_1} + \frac{v}{R_2}.\text{)}$$

The above equation says that two resistances interconnected in parallel is equivalent to a single resistance having a value equal to the product of two resistances divided by their sum, $\boxed{R_{Eq} = \frac{R_1 R_2}{R_1 + R_2}}$. Note that the equivalent resistance of two positive resistances in parallel is less than either of the two resistances.



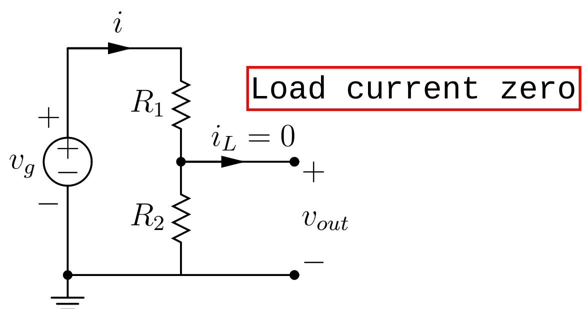


Principles of Electrical Engineering I
Voltage divider circuit with or without load

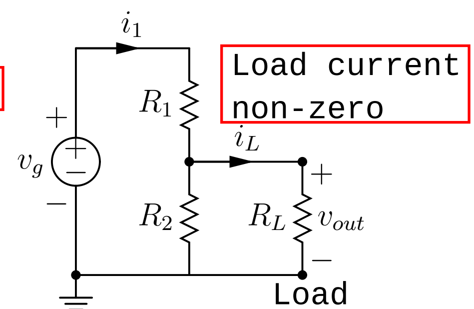
Consider the circuit shown below on the left side. It is easy to see that

$$i = \frac{v_g}{R_1 + R_2} \quad \text{and} \quad v_{out} = iR_2 = \frac{R_2}{R_1 + R_2} v_g.$$

Since $\frac{R_2}{R_1+R_2} < 1$, the output voltage v_{out} is a fraction of input voltage v_g. Thus the circuit is a voltage divider circuit. In fact in this case the voltage v_g is divided between R_1 and R_2. The circuit shown on the left is an ideal circuit where the current i_L drawn from the circuit is zero. Often, a load represented by the resistance R_L is connected across R_2, and thus i_L is non-zero.



Voltage Divider with no load, $R_L = \infty$



Voltage Divider with load R_L

Let us next determine the output voltage v_{out} when the load R_L is present. In this case, R_L and R_2 are in parallel (normally, denoted by $R_L \,||\, R_2$). Show that

$$v_{out} = \frac{R_2}{R_1 + R_2 + \frac{R_1 R_2}{R_L}} v_g.$$

It is easy to see that the output voltage v_{out} when the load R_L is present is smaller than when R_L is not present (meaning when $R_L = \infty$).

$$i_1 = \frac{v_g}{R_1 + R_L \,||\, R_2}$$

$$\begin{aligned} v_{out} &= i_1(R_L \,||\, R_2) \\ &= \frac{R_L \,||\, R_2}{R_1 + R_L \,||\, R_2} v_g \\ &= \frac{\frac{R_L R_2}{R_L + R_2}}{R_1 + \frac{R_L R_2}{R_L + R_2}} v_g \\ &= \frac{R_L R_2}{R_1 R_L + R_1 R_2 + R_L R_2} v_g \\ &= \frac{R_2}{R_1 + R_2 + \frac{R_1 R_2}{R_L}} v_g. \end{aligned}$$

A design problem: As we said earlier, **the backbone of any design is 'thorough analysis' of the problem or circuit at hand.** We have analyzed above the voltage divider circuit. Utilizing the above analysis, determine R_1 and R_2 of a voltage divider circuit to satisfy the following specification:

Specification: An automobile chassis display light having a resistance of $400\,\Omega$ requires for its proper operation 9 V with a tolerance of $\pm 5\%$. Design a voltage divider circuit that gets its supply voltage from the automobile battery, and feeds its output to the chassis display light. We are also told that the output voltage of the voltage divider should be at 9 V with a tolerance of $\pm 5\%$ whether the chassis display light is on or off.

Hint: Note that v_g is the battery voltage, and hence $v_g = 12$ V. We need to determine R_1 and R_2. Take the voltage v_{out} of the voltage divider circuit under no load (when the chassis display light is switched off) as $9 + (0.05)9 = 9.45$ Volts. This gives one equation between R_1 and R_2. Next, take the voltage v_{out} of the voltage divider circuit under load (when the chassis display light is switched on) as $9 - (0.05)9 = 8.55$ Volts. This gives another equation between R_1 and R_2. Solve the resulting two equations for R_1 and R_2.

Solution: $R_1 = 53.6\Omega$ and $R_2 = 198.52\Omega$.

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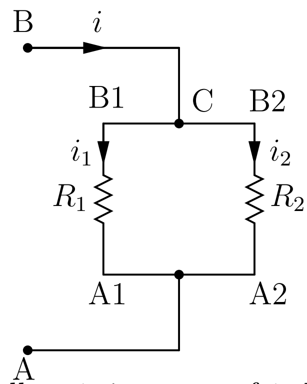
Current division rule: Consider two resistances R_1 and R_2 in parallel as shown, and an input current i as shown.



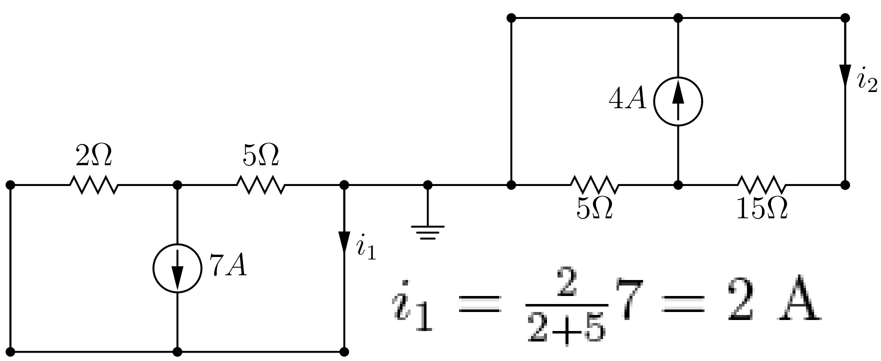
Determine i_1 as well as i_2 in terms of i, R_1, and R_2.
KCL equation at the node C yields
$\boxed{i = i_1 + i_2}$.
KVL equation around the loop A1B1B2A2A1 yields
$\boxed{i_1 R_1 = i_2 R_2}$.
By solving the above equations, we get

$$i_1 = \frac{iR_2}{R_1 + R_2},$$

and

$$i_2 = \frac{iR_1}{R_1 + R_2}.$$

Problem: By utilizing appropriately the current division rule, determine the currents i_1 and i_2.



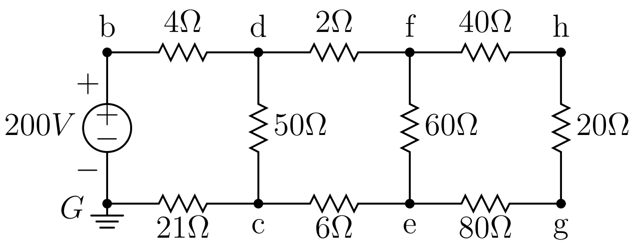
$$i_1 = \frac{2}{2+5} 7 = 2 \text{ A}$$

Hint Observe that the resistances 5Ω and 15Ω are in parallel with the current source $4A$. Similar observation can be made regarding the other part of the circuit.

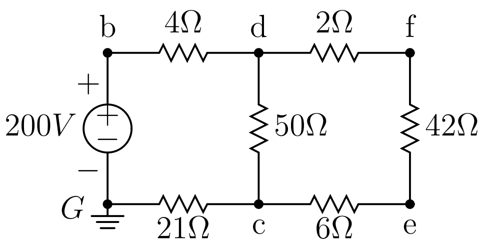
Principles of Electrical Engineering I

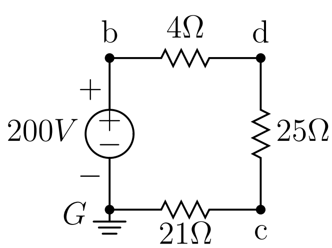
Series-Parallel Resistances – Example

Solve the following circuit to determine the current in each branch.

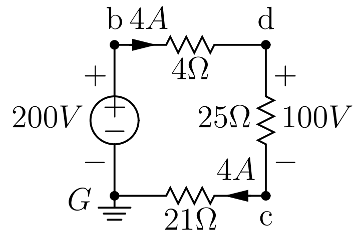


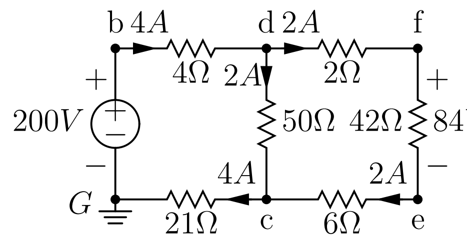
We systematically simplify the above circuit as shown below.

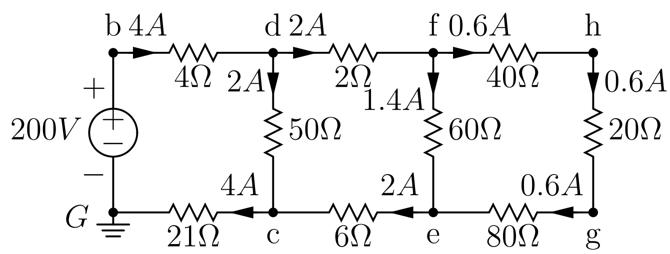




It is easy to solve the above circuit. The solution is depicted in the first figure on the right side column.







Example: Our interest in this problem is to determine the currents i_x and i_y in the circuit shown in Figure 1. For this purpose we simplify the circuit as much as we can, solve the simple circuit and then come back to solve the given circuit.

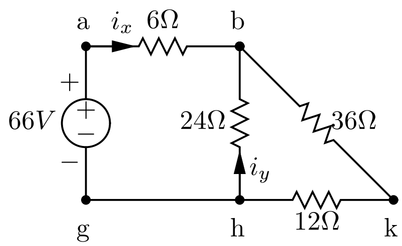


Figure 1

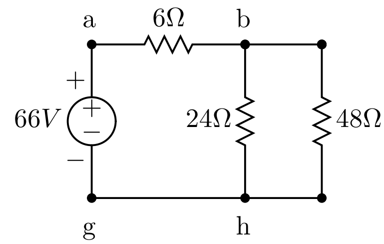


Figure 2

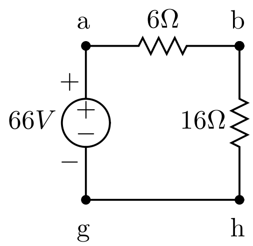


Figure 3

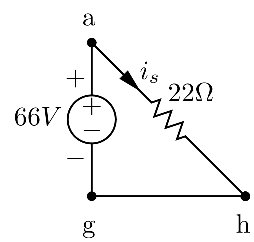


Figure 4

Note that i_s is the current that is supplied by the 66V source.
By Ohm's law, $i_s = 3$ A.
In terms of the circuit of Figure 1, i_s is indeed i_x. Thus $i_x = 3$ A.

In the above analysis, we successively simplified the given circuit into simpler circuits. Finally, we solved a simple circuit as in Figure 4. To determine other currents in the previous circuits, we need to trace back from the simple circuit of Figure 4 to the circuit of Figure 3, and so on. We do this below.

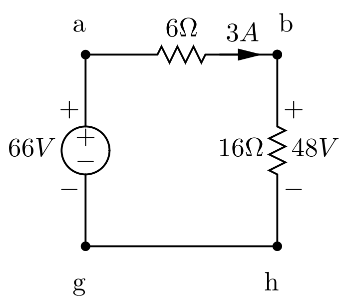


Figure 3 re-drawn

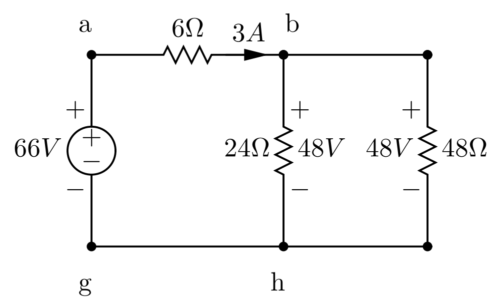


Figure 2 re-drawn

We can evaluate the current from node b to node h via 24Ω in two ways. Knowing the voltage across 24Ω as 48 V, we note the current from node b to node h as 2 A by simple Ohm's law. Or we can evaluate it by current division rule by knowing the current 3 A coming into node b splits into two parts, one part going through 24Ω and the other part going through 48Ω. This enables us to determine all currents in the circuit of Figure 1 as shown on the right. Finally, we note that i_y we were asked to determine is -2 A.

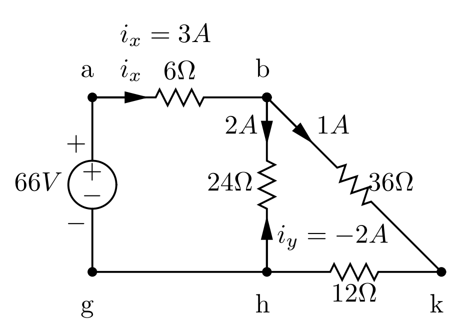


Figure 1 re-drawn

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Let $i_d = 1$ A. Determine the currents and voltages marked on the circuit of Figure 1. All the voltages of nodes marked in circles are with respect to the ground G. Start on the right hand side. Compute the variables in the following order by writing appropriate KCL and KVL equations,

$$v_4, \quad v_3, \quad i_c, \quad i_3, \quad v_2,$$

$$i_b, \quad i_2, \quad v_1, \quad i_1, \quad \text{and } v_g.$$

Answers: $v_g = 27$ V, $v_1 = 27$ V, $v_2 = 9$ V, $v_3 = 3$ V, and $v_4 = 1$ V.
$i_1 = 9$ A, $i_2 = 9$ A, and $i_3 = 3$ A.
$i_b = 6$ A, and $i_c = 2$ A.

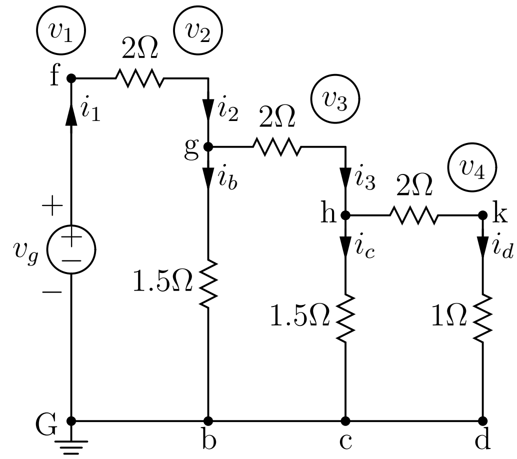

Figure 1

It is obvious that $v_4 = 1$ V since it is the voltage across 1Ω carrying a current of 1 A. The 2Ω and 1Ω in the path hkd have the same current of 1 A, and thus are in series. This implies that the voltage v_3 which is the voltage rise from d to k to h equals 3 V. This voltage v_3 is also the voltage rise from c to h. Hence $i_c = \frac{v_3}{1.5} = \frac{3}{1.5} = 2$ A. Then the KCL at the node h results in $i_3 = i_c + i_d = 3$ A. This in turn implies that the voltage rise from h to g is $i_3 \times 2 = (3)2 = 6$ V. We can now compute v_2 which is the voltage rise from c to h to g as $6 + 3 = 9$ V. This voltage v_2 is also the voltage rise from b to g. Hence $i_b = \frac{v_2}{1.5} = \frac{9}{1.5} = 6$ A. Then the KCL at the node g results in $i_2 = i_b + i_3 = 9$ A. This in turn implies that the voltage rise from g to f is $i_2 \times 2 = (9)2 = 18$ V. We can now compute v_1 which is the voltage rise from b to g to f as $18 + 9 = 27$ V. It is obvious that $v_g = v_1 = 27$ V.

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$\boxed{\textbf{Theme Example}}$ – Digital to Analog Converter (DAC)

Keep this pages in your active file. You need to refer to this page in several Home-work problems.

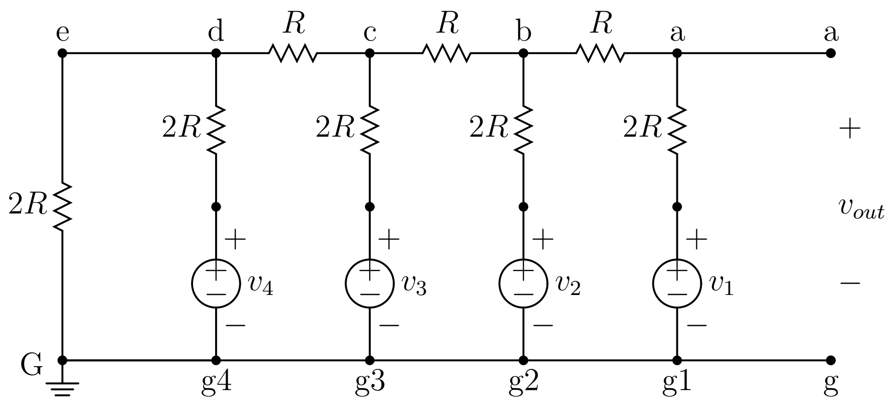


Figure 1

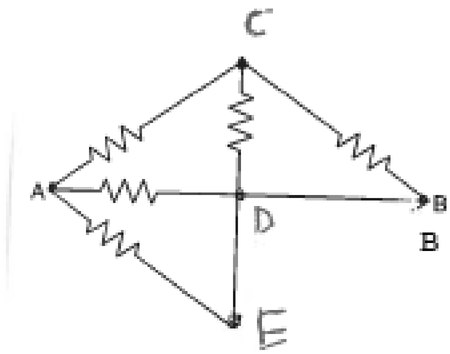
Often one requires a **digital to analog converter (DAC)**. It is a signal processing circuit. There exist a number of such circuits. Figure shows a four bit DAC circuit known as R-$2R$ ladder circuit. Here v_1, v_2, v_3, and v_4 are the voltages corresponding to most significant bit (MSB), the next MSB, the next MSB, and the least significant bit (LSB). Whenever any bit is **on**, it has a positive voltage (say, 5 V); and whenever any bit is **off**, it has a zero voltage. The DAC circuit must add all the voltages by weighing the bits appropriately. The relative weights given to the MSB, the next MSB, the next MSB, and the least significant bit (LSB) must be 8, 4, 2, and 1. Since the weights alloted are relative, one can for example weigh them in the order of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$, where the weight $\frac{1}{2}$ corresponding to the MSB. We can show that the circuit shown generates an output voltage

$$v_{out} = \frac{v_1}{2} + \frac{v_2}{4} + \frac{v_3}{8} + \frac{v_4}{16}.$$

In this course, we will analyze this circuit in **a number of ways**. Several Home-Work problems use this circuit as a **theme problem**. Although one might be interested only in the output voltage v_{out}, we will determine all branch currents as well as other variables of interest.

Example:

Assume all the resistances in the circuit given below are one ohm each. Determine the equivalent resistance between the terminals (1) A and B, (2) A and C, (3) A and D, (4) B and C, (5)B and D, (6) B and E, (7) C and D.



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Theme Example – DAC – Superposition Method

Digital to analog converter (DAC) circuit is shown on the right.

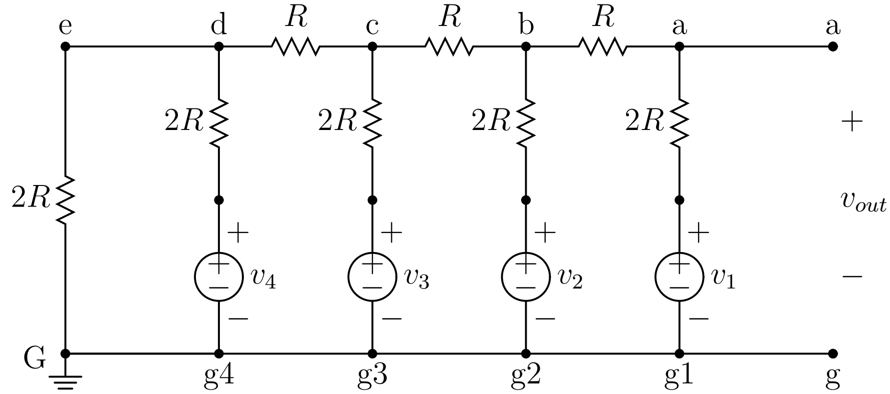


Figure 1

We plan to solve the DAC circuit by utilizing what is known as a **superposition theorem**. For linear circuits, the superposition theorem states the following:
Suppose only one source is kept in the circuit and all other sources are set to zero. The circuit then simplifies and one can solve for any particular variable x of the circuit. The value of x thus obtained is the effect on x of the source that is kept in the circuit. One can repeat determining x as often as needed while at each time keeping only some different source and setting all other sources to zero. Then, the net value of x is the algebraic sum of all such individually determined values of x.
Source v_2 only: Suppose we like to determine all node voltages v_a, v_b, v_c, and v_d due to the source v_2 only. The node voltages are measured with respect to G. Then, the circuit with the presence of v_2 but all other sources set to zero is given by Figure 2 where several voltage and current variables are marked.

Parallel equivalent of this is R as shown on next page in Figure 2A

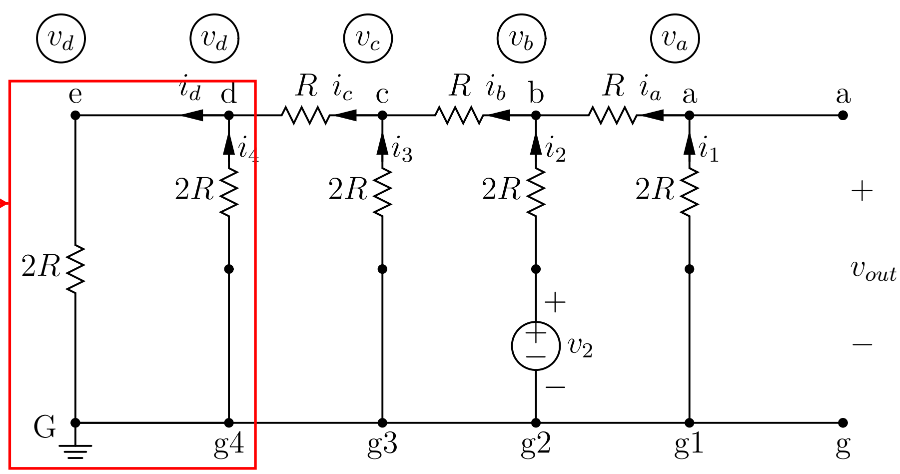


Figure 2

If we solve the above circuit correctly, the solution for node voltages v_a, v_b, v_c, and v_d is given by

$$v_a = \frac{1}{4}v_2, \quad v_b = \frac{3}{8}v_2, \quad v_c = \frac{3}{16}v_2, \quad v_d = \frac{3}{32}v_2.$$

We proceed now to solve the circuit of Figure 2 by using series-parallel resistance equivalents that would simplify the circuit. For clarity, we simplify the circuit into several circuits in succession and then solve each of the simplified circuits. We show below all the steps involved.

Step 1: The two parallel resistances $2R$ each, one between the terminals 'e' and 'G' and the other between the terminals 'd' and 'g4', are combined into a single resistance of value R. This simplication reduces the circuit of Figure 2 to that of circuit of Figure 2A.

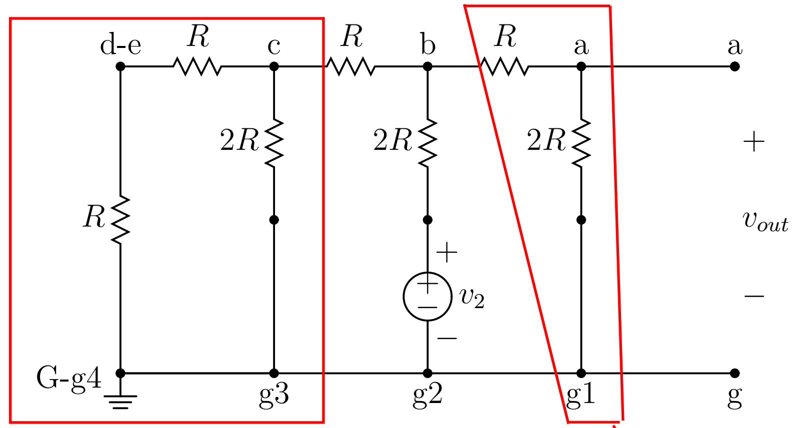


Figure 2A

Series parallel equivalent

Series equivalent

Step 2: The two series resistances R each, one between the terminals 'd-e' and 'G-g4' and the other between the terminals 'c' and 'd-e', can be combined into a single resistance of value $2R$. This resistance of value $2R$ is in parallel with another resistance of value $2R$ which is between the terminals 'c' and 'g3'. This parallel combination yields a single resistance of value R. Also, the two series resistances, one between the terminals 'b' and 'a' having the value R and the other between the terminals 'a' and 'g1' having the value $2R$, can be combined into a single resistance of value $3R$. This simplication reduces the circuit of Figure 2A to that of circuit of Figure 2B. By doing so, the terminals 'd-e' and 'a' are eliminated.

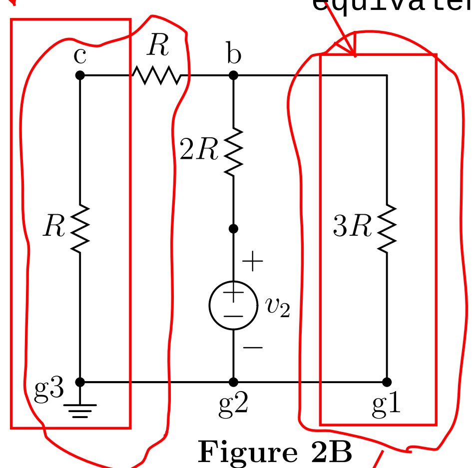


Figure 2B

Series equivalent between b and g3 and then parallel equivalent

Step 3: The two series resistances R each, one between the terminals 'c' and 'g3' and the other between the terminals 'b' and 'c', can be combined into a single resistance of value $2R$. This resistance of value $2R$ is in parallel with another resistance of value $3R$ which is in the path 'bg1g2'. This parallel combination yields a single resistance of value $R1 = \frac{6}{5}R$. This simplication reduces the circuit of Figure 2B to that of circuit of Figure 2C. By doing so, the terminals 'c' is eliminated.

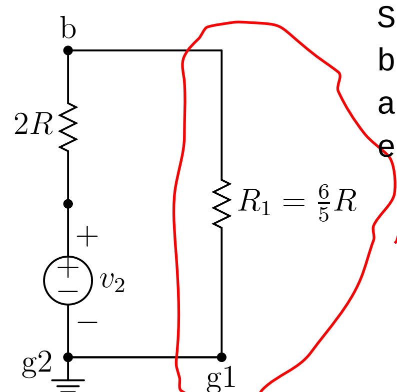


Figure 2C

Step 4: We can easily solve the circuit of Figure 2C. The resistances $2R$ and R_1 are in series and hence the net resistance equals $\frac{16}{5}R$. Thus, by Ohm's law

$$i_2 = \frac{v_2}{\frac{16}{5}R} = \frac{5v_2}{16R}.$$

We note that

$$v_b = v_2 - i_2 2R = v_2 - \frac{5v_2}{16R}2R = \frac{3}{8}v_2.$$

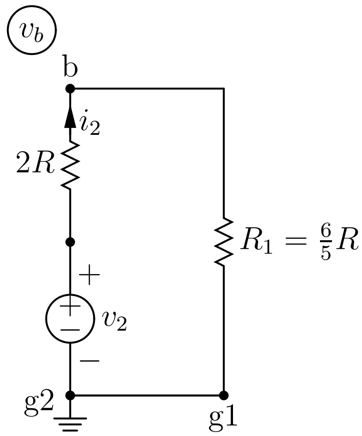


Figure 2C

Step 5: By knowing v_b and i_2, we can move on to solve the circuit of Figure 2B. Before we do so, let us look at the circuit of Figure 2D. We note that by Ohm's law,

$$i_b = \frac{v_b}{2R} = \frac{3}{16R}v_2.$$

Also,

$$\begin{aligned} v_c &= i_b R = \frac{3}{16}v_2, \\ i_x &= \frac{v_b}{3R}. \end{aligned}$$

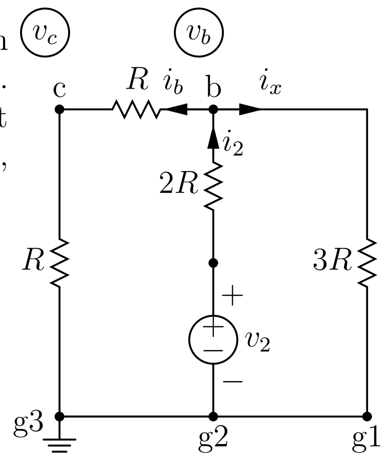


Figure 2D

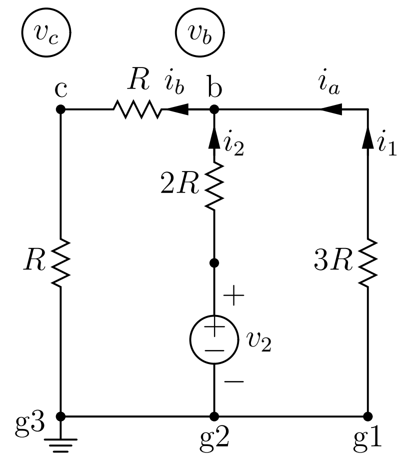


Figure 2B

What we want to determine is not i_x, but i_1 and i_a in Figure 2B. However, it is easy to note that i_1 and i_a are both in opposite direction to i_x,

$$i_1 = i_a = -i_x = \frac{-v_b}{3R} = -\frac{1}{8R}v_2.$$

Step 6: We can move on to solve the circuit of Figure 2A. We note that by Ohm's law,

$$i_c = \frac{v_c}{2R} = \frac{3}{32R}v_2, \quad \text{and} \quad i_y = \frac{v_c}{2R} = \frac{3}{32R}v_2.$$

$$v_a = -i_1 2R = \frac{1}{4}v_2 \quad \text{and} \quad v_d = i_c R = \frac{3}{32}v_2.$$

Moreover, we note that

$$i_3 = -i_y = \frac{-v_c}{2R} = -\frac{3}{32R}v_2.$$

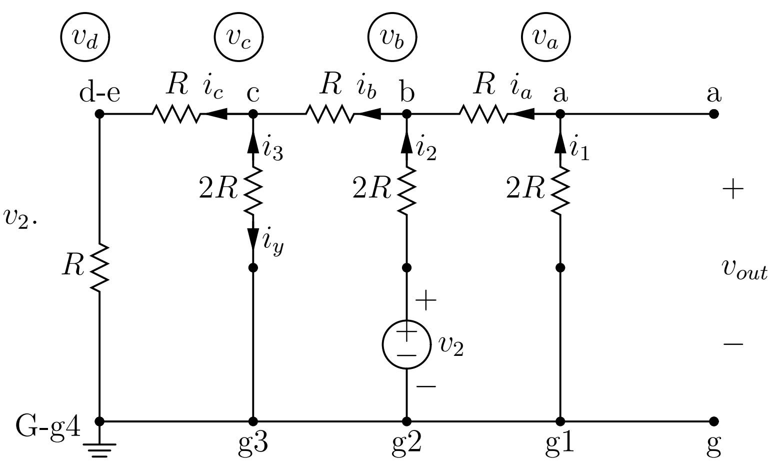


Figure 2A

We can now move on to the circuit of Figure 2 in order to obtain the remaining variables. We note that

$$i_d = \frac{v_d}{2R} = \frac{3}{64R}v_2 \quad \text{and} \quad i_4 = \frac{-v_d}{2R} = -\frac{3}{64R}v_2.$$

Can you justify why negative sign is used in computing i_4? Think of sign convention in Ohm's law.

Name in CAPITAL LETTERS:
LAST FOUR DIGITS OF ID NUMBER:

An email will be sent indicating due date.

HW: Home Security

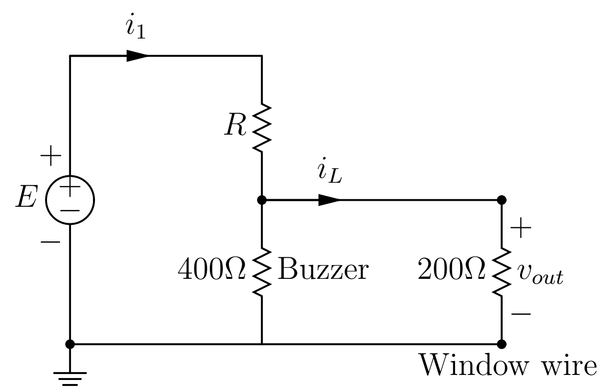
332:221 Principles of Electrical Engineering I

This design problem is a home-work which will be collected and graded.

A residential and commercial building security company hired you as an engineer. You have been asked to design a circuit that will enable a buzzer to sound whenever a window is broken. The components that are provided for this task are as follows:

1. One long 200 Ω wire that can be attached to the window using a non-conductive epoxy. Whenever a window breaks, the wire breaks as well.
2. A buzzer having a resistance of 400 Ω and sounds an alarm when a DC voltage applied to it is 6 V or more; however the buzzer burns up if the voltage applied to it exceeds 24 V.
3. One 9 V battery and twelve 1.5 V AAA size batteries that are rechargeable. Assume that the batteries do not have any internal resistance.
4. Ten 100 Ω resistors, twenty 50 Ω resistors, and one hundred one Ω resistors.

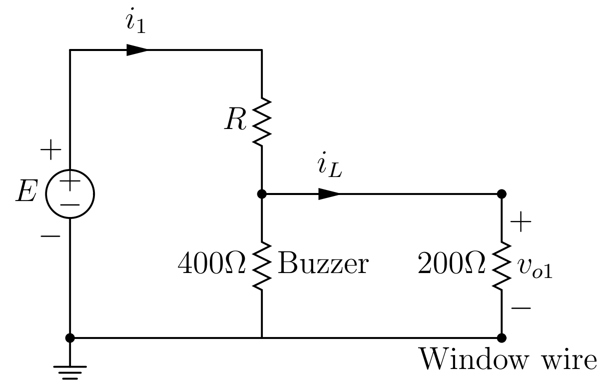
An experienced engineer tells you that one can use a voltage divider circuit with the 200 Ω window wire as the load connected across the buzzer as shown on the right with E and R yet to be decided. The reason such a circuit could work with E and R properly selected is this: the combined parallel equivalent resistance of the 200 Ω window wire and the buzzer is smaller than 200 Ω when the window wire is not broken, and is 400 Ω (buzzer resistance) when the window wire is broken. Whenever the window is broken, an increase in resistance of one of the two legs of a basic voltage divider circuit can establish a higher voltage across it.



Follow the steps given in the next page in order to determine the values for E and R to meet the specifications.

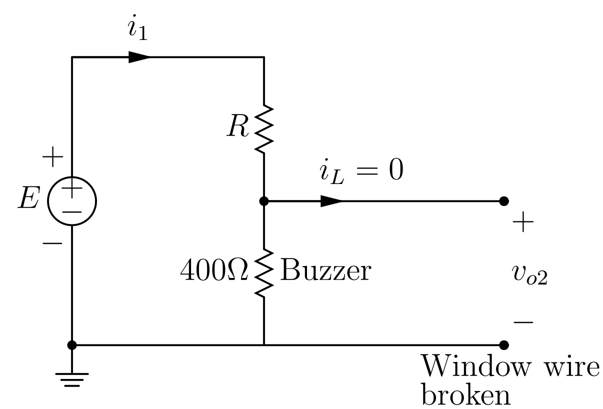
Analysis with window not broken:

When the window is not broken, the voltage divider circuit is as shown on the right. For clarity and for comparison with the next case, we have denoted the output as v_{o1}. Derive a relationship between the output v_{o1} and the unknown variables E and R. We note that one of the design constraints is that v_{o1} should be less than 6 V. This yields one constraint equation. Write this equation in the space below.



Analysis with window broken:

When the window is broken, the voltage divider circuit simplifies to the one shown on the right. In this case, we have denoted the output as v_{o2}. Derive a relationship between the output v_{o2} and the unknown variables E and R. We note that one of the design constraints is that v_{o2} should be greater than 6 V but less than 24 V. This yields another constraint equation. Write this equation in the space below.



Select the values for E and R to satisfy the above two constraint equations, and verify that the chosen values indeed will work. Do all your work in the space below, and backside if needed.

It is helpful to do it to learn all aspects of series-parallel combinations.

HW: DAC v_3 only

Name in CAPITAL LETTERS:
LAST FOUR DIGITS OF ID NUMBER:

332:221 Principles of Electrical Engineering I

Theme Example – DAC– Source v_3 only: HW, ~~collected and graded~~

The following circuit is the Digital to Analog Converter (DAC) where v_1, v_2, and v_4 are set to zero. This implies that only the source v_3 exists. Determine all node voltages v_a, v_b, v_c, and v_d due to the source v_3 only. Use the method of simplifying the circuit by series-parallel equivalents as illustrated in the class when only source v_2 exists. Follow systematically the guide lines given below:

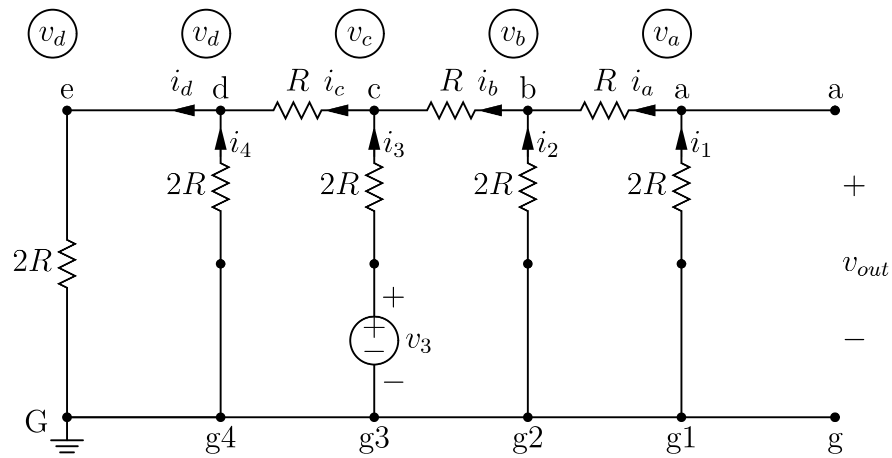


Figure 1

Step 1: So far as the terminals 'g_3' and 'c' are concerned, one can simplify the circuit of Figure 1 to the circuit of Figure 2. Determine the appropriate values for R_1 and R_2.

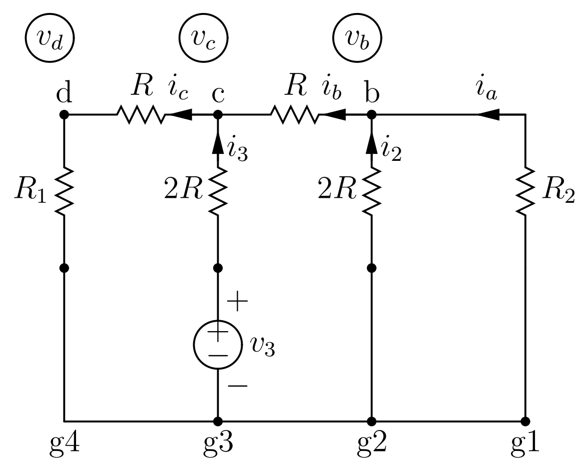


Figure 2

Step 2: So far as the terminals 'g_3' and 'c' are concerned, one can simplify the circuit of Figure 2 to the circuit of Figure 3. Determine the appropriate values for R_3 and R_4.

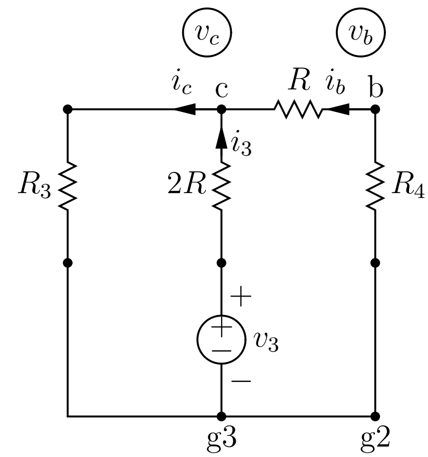


Figure 3

Step 3: So far as the terminals 'g_3' and 'c' are concerned, one can simplify the circuit of Figure 3 to the circuit of Figure 4. Determine the appropriate value for R_5.

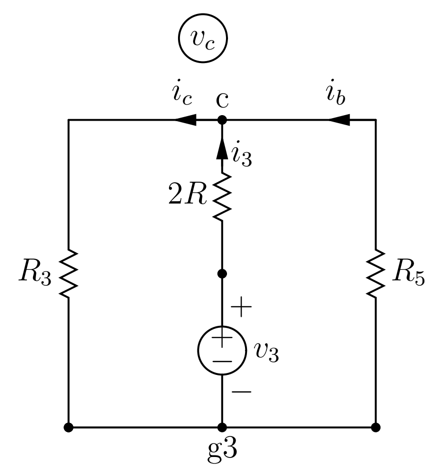


Figure 4

Step 4: So far as the terminals 'g_3' and 'c' are concerned, one can simplify the circuit of Figure 4 to the circuit of Figure 5. Determine the appropriate value for R_6.

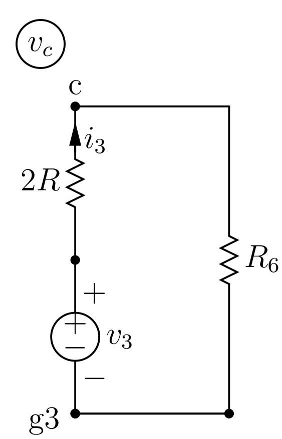


Figure 5

Step 5: Solve for i_3 in view of the circuit of Figure 5. Once i_3 is determined, solve for v_c.

Step 6: Once v_c is determined, consider the circuit of Figure 4 and determine i_b and i_c.

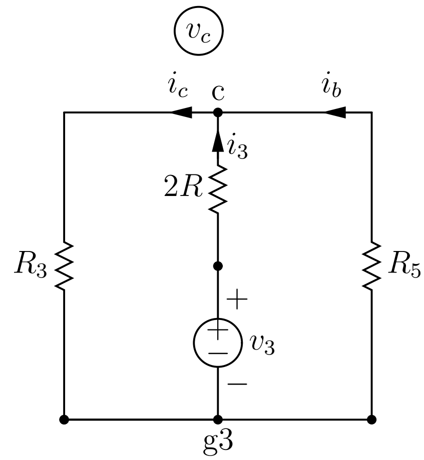


Figure 4

Step 7: Once v_c, i_b, and i_c are known, consider the circuit of Figure 3 and determine v_b.

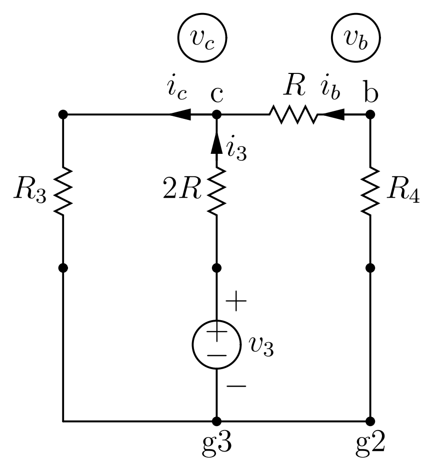


Figure 3

Step 7: Once v_b, v_c, i_b, and i_c are known, consider the circuit of Figure 2 and determine i_a and v_d.

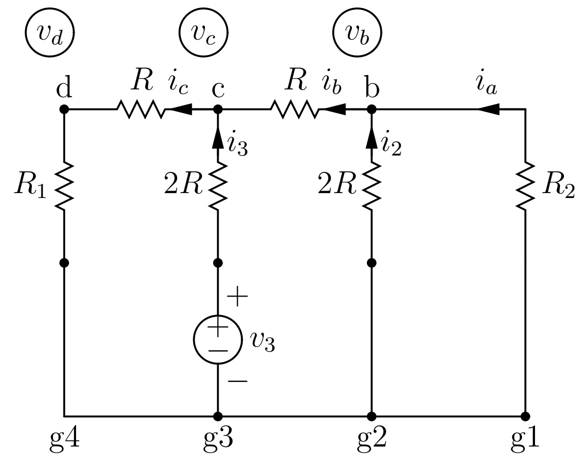


Figure 2

Step 8: Finally determinev_a from the circuit of Figure 1. Mark clearly all the values of current and voltage variables on the circuit of Figure 1 given below or write them down in the space below.

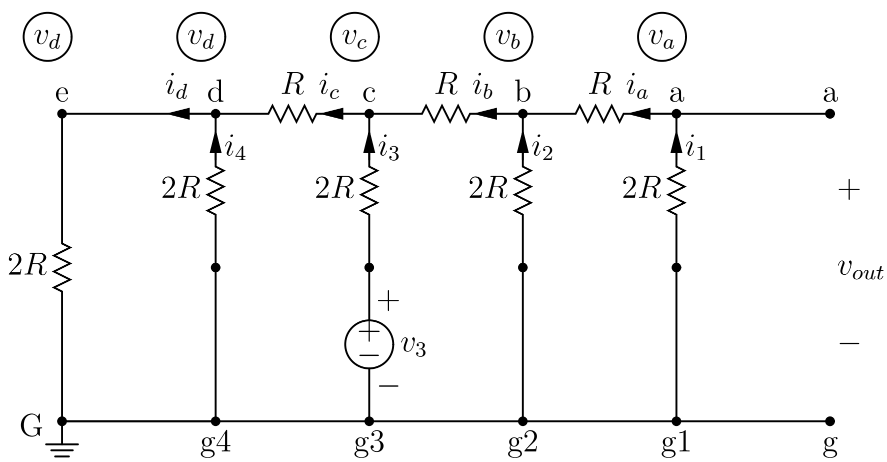


Figure 1

$v_a =$ $\quad$ $v_b =$ $\quad$ $v_c =$ $\quad$ $v_d =$

$i_a =$ $\quad$ $i_b =$ $\quad$ $i_c =$ $\quad$ $i_d =$

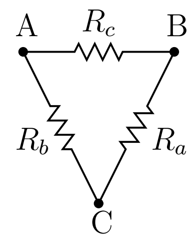
$i_1 =$ $\quad$ $i_2 =$ $\quad$ $i_3 =$ $\quad$ $i_4 =$

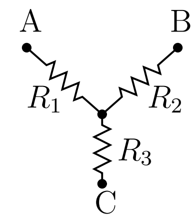
$\Delta-$ Y equivalents

$$R_1 = \frac{R_b R_c}{R_a+R_b+R_c} \qquad R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1}$$

$$R_2 = \frac{R_a R_c}{R_a+R_b+R_c} \qquad R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2}$$

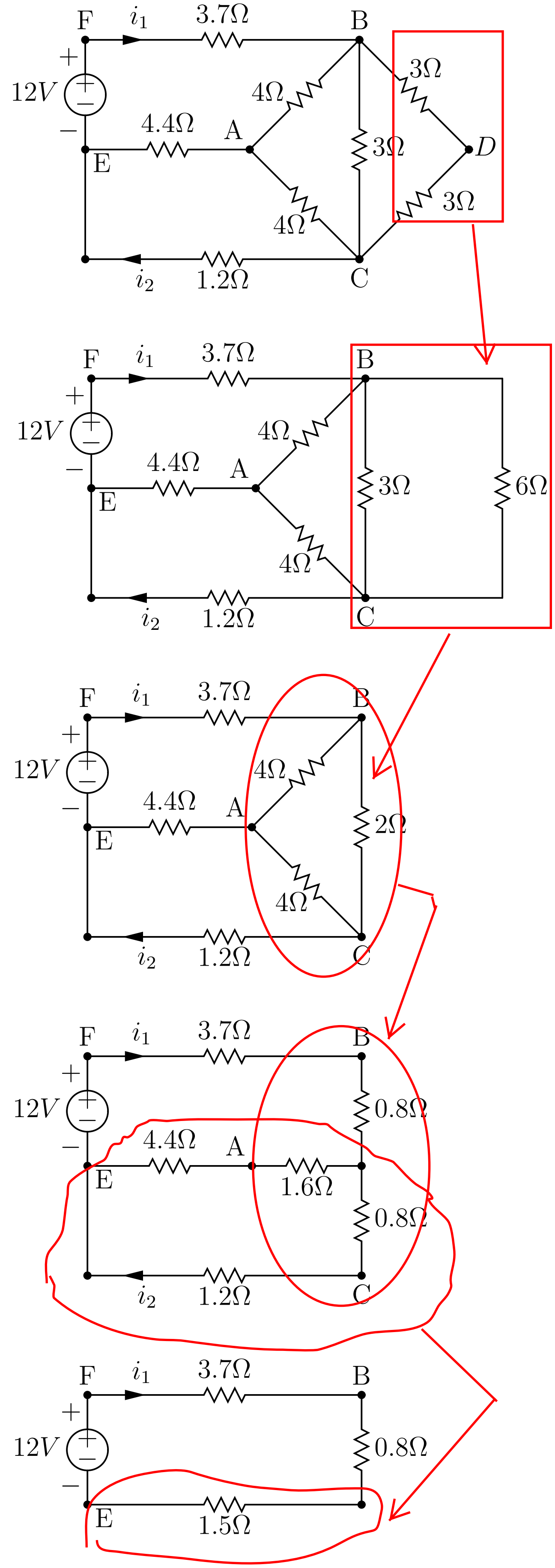
$$R_3 = \frac{R_a R_b}{R_a+R_b+R_c} \qquad R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3}$$





Problem 5: (30 points)

A number of circuits are given on the right. The top most circuit is the starting circuit, and all the other circuits are equivalent circuits to the top most circuit with respect to the terminals E and F. The equivalent circuits are constructed progressively by recognizing series, parallel, and Δ to Y equivalents of appropriate combinations of resistances. Recognize such series, parallel, and Δ to Y equivalents, and mark on the circuits appropriate values of all unmarked resistances in each circuit. Determine the value of current i_1 with the help of the last circuit, and then move up to the previous circuit to determine the value of current i_2 (Use current division rule).



Solution:

We have the following steps.

1. Recognize that the two 3Ω resistances in the path BDC are in series. Thus they can be replaced equivalently by a 6Ω resistance. This enables us to obtain the equivalent circuit shown in the second circuit.

2. Recognize that the two resistances 3Ω and 6Ω between the terminals B and C are in parallel. Combining them yields 2Ω as displayed in the equivalent circuit shown in the third circuit.

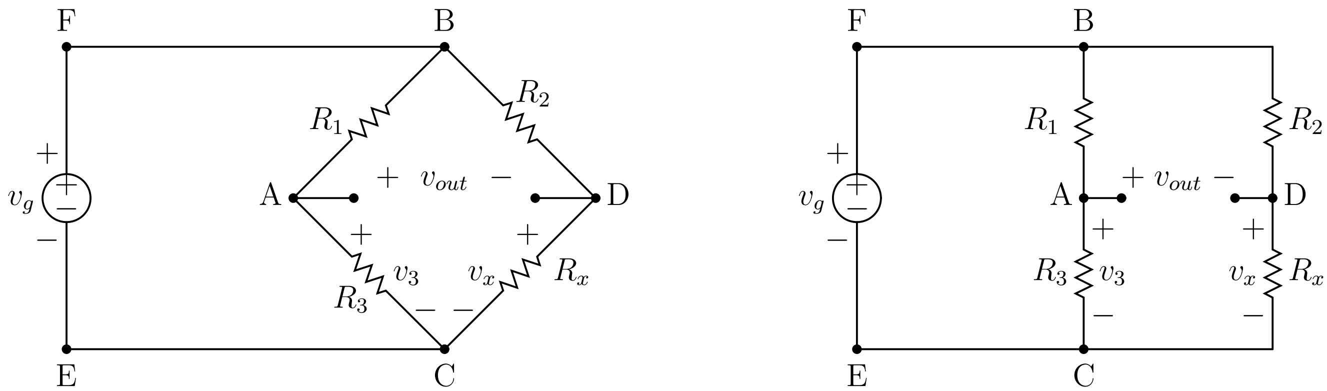
3. Recognize that the three resistances 4Ω, 4Ω, and 2Ω between the terminals A, B, and C form a Δ. Transforming the Δ to Y yields the equivalent circuit shown in the fourth circuit.

4. Recognize that the two resistances 4.4Ω and 1.6Ω are in series. Also, recognize that the two resistances 1.2Ω and 0.8Ω are in series. Moreover, both of the resulting series equivalents are in parallel. This yields the last equivalent circuit.

5. In the last equivalent circuit, all the resistances are in series, and thus we can easily calculate i_1 as 2 Amps.

6. Finally, we can calculate i_2 by current division rule as $\frac{6}{6+2} i_1 = 1.5$ Amps

Problem 4: (20 points) The so called Wheatstone bridge circuit is often drawn as displayed below in two different layouts which are electrically equivalent. Note that we may think of Wheatstone bridge circuit as two voltage divider circuits put together, one voltage divider circuit is CEFBAC and the other is CEFBDC. Assume that $v_g = 100$ V.



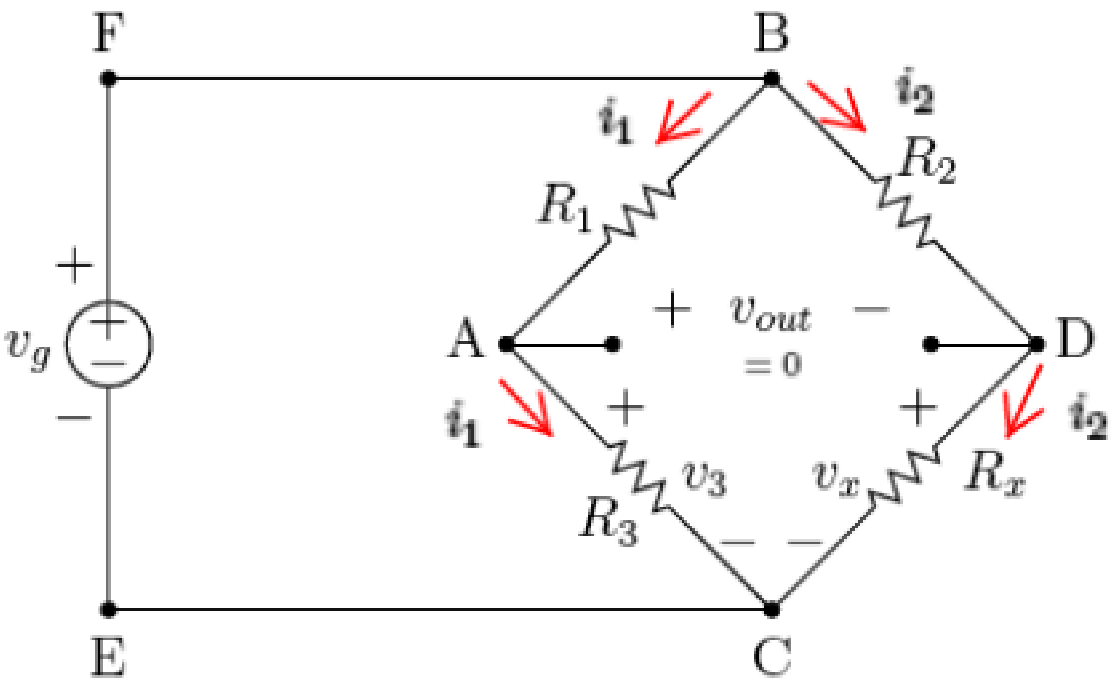
1. Assume that $R_1 = R_3 = 100\Omega$, and determine the voltage v_3 (use voltage divider principle).

2. Assume that $R_2 = 100\Omega$ and $R_x = 300\Omega$, and determine the voltage v_x (use voltage divider principle).

3. Determine the voltage $v_{out} = v_3 - v_x$ by utilizing the results of above two steps.

4. Suppose $R_1 = R_2 = R_3 = 100\Omega$, determine R_x so that we have $v_x = 20$ V. What is the value of v_{out} in this case.

Measuring an unknown Resistance
by Wheatstone-bridge



Adjust any one of the resistances, R_1 or R_2 or R_3 until $v_{out} = 0$

$$i_1 R_1 = i_2 R_2$$

$$i_1 R_3 = i_2 R_x$$

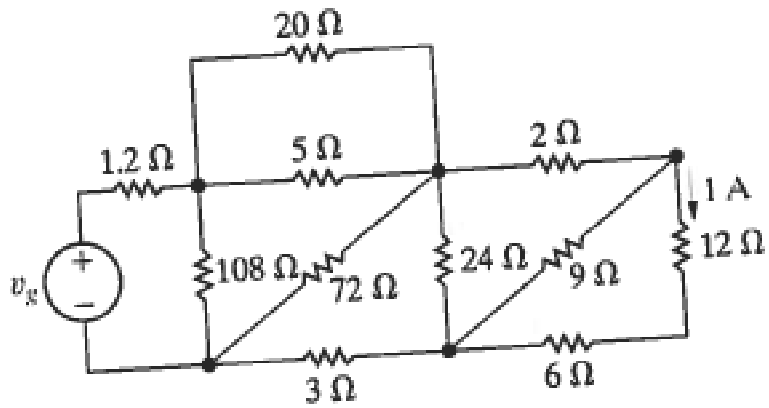
Dividing one equation with the other,

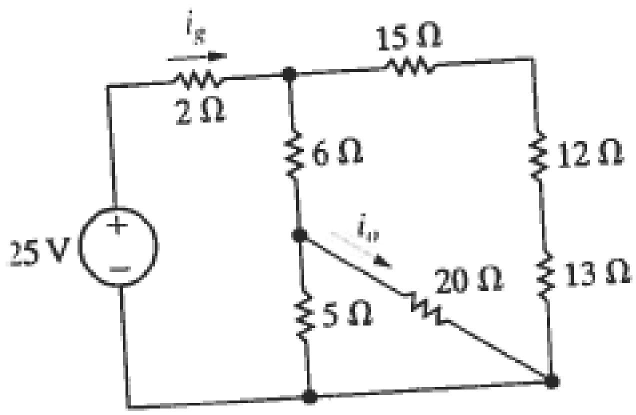
$$\frac{i_1 R_1}{i_1 R_3} = \frac{i_2 R_2}{i_2 R_x}$$

$$\frac{R_1}{R_3} = \frac{R_2}{R_x}$$

$$R_x = \frac{R_2 R_3}{R_1}$$

Solve the following circuits
by simplifying them as much
as possible and then moving back
to determine the variables in the
given circuit.





Let $i_d = 1$ A. Determine the currents and voltages marked on the circuit of Figure 1. All the voltages of nodes marked in circles are with respect to the ground G. Start on the right hand side. Compute the variables in the following order by writing appropriate KCL and KVL equations,

$$v_4, \quad v_3, \quad i_c, \quad i_3, \quad v_2,$$

$$i_b, \quad i_2, \quad v_1, \quad i_1, \quad \text{and } v_g.$$

Answers: $v_g = 125$ V, $v_1 = 125$ V, $v_2 = 25$ V, $v_3 = 5$ V, and $v_4 = 1$ V.
$i_1 = 25$ A, $i_2 = 25$ A, and $i_3 = 5$ A.
$i_b = 20$ A, $i_c = 4$ A, and $i_d = 1$ A.

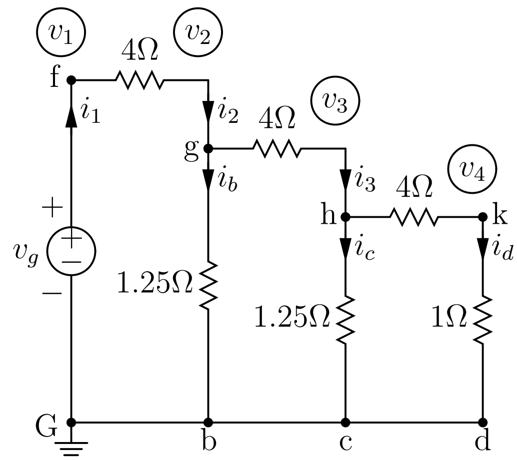


Figure 1

Show that the equivalent resistance between the terminals is 90 ohms