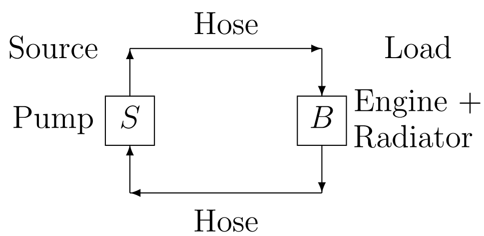
332:221 Principles of Electrical Engineering I

Radiator Cooling System

Radiator Cooling System: Consider the components,

1. Water pump
2. Hoses
3. Engine + Radiator.

These components are interconnected to form a closed path (circuit).



Pump exerts pressure or force on water.

Water circulates around the closed path.

What does the rate of flow of water (water current) depend on?

What are the variables of circuit (closed path)?

1. Force
2. Water current
3. Power
4. Work done (Energy)

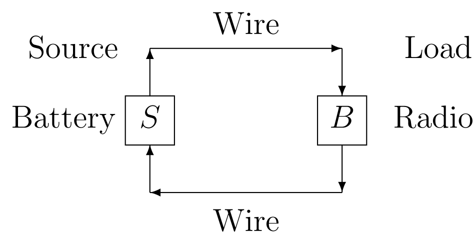
Each component has a pressure difference between its ends or terminals.
Each component has a current of water through it.
Each component either generates or consumes power
Which component generates power?
Which component consumess power?

A simple Electric Circuit

A simple circuit: Consider the components,

1. Battery
2. Wires
3. Radio.

These components are interconnected to form a closed path (circuit).



Battery exerts pressure or force on electric charge.
Electric charge circulates around the closed path.
What does the rate of flow of charge (called current) depend on?

What are the variables of circuit (closed path)?

1. Electric Force (Voltage source)
2. current
3. Power
4. Work done (Energy)

Each component has an electrical potential difference (voltage) across its terminals.
Each component has a current (current of charge) through it.
Each component either generates or consumes power
Which component generates power?
Which component consumess power?

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Preview – Circuit Variables and Components

Analogy: When you start reading a novel, until you understand the characters involved and their nature, the theme of the story is not clear. However, once you know the characters, the story flows easily.

Our Story is Circuit Analysis

What is a circuit? What is Circuit Analysis?

A circuit is an interconnection of components. forming a closed path or many closed paths.

A component is like a person or a character in a novel. Interaction between characters makes a story. Interaction between components makes the circuit behave the way it behaves.

Each character in a novel has his/her own personality. Each component in a circuit has its own behavior.

Behavior of a component is exhibited by its variables. voltage across it and current through it.

What are circuit or component variables?

Charge q **or** Q

Voltage v **or** V

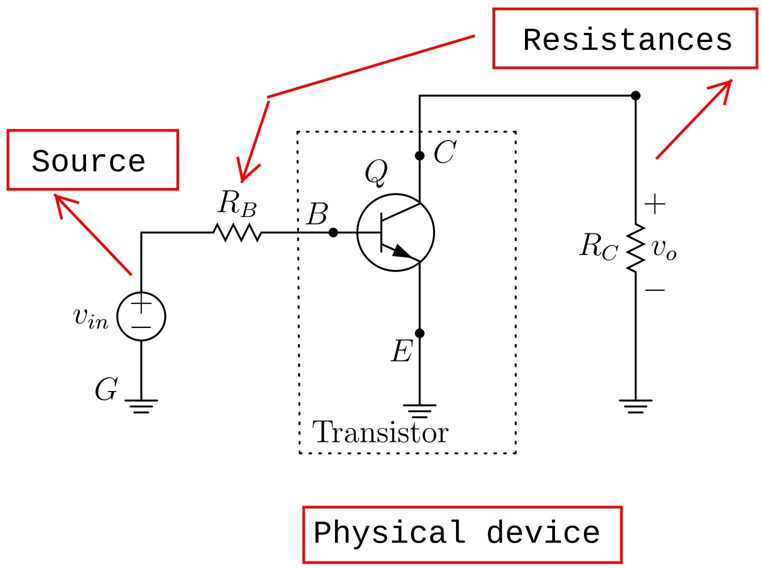
Current i **or** I

Power p

Energy or Work done W

We will discuss each of these systematically soon.
Each component has a voltage across it and current through it.

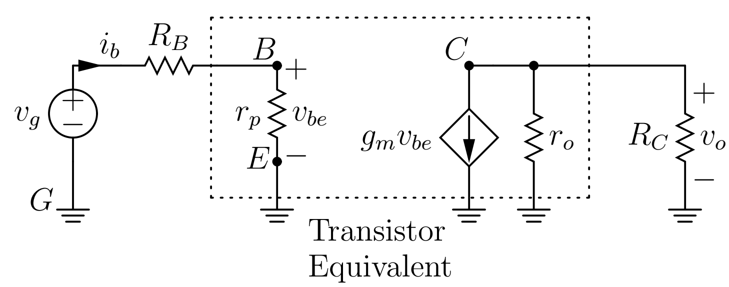
The following is just a sketchy discussion of a circuit. This is given here just to get a feel for a circuit and its components.



Circuit Elements:
1. v_{in}= Independent Voltage Source (battery), a two terminal element.
2. Transistor A physical device, a three terminal element.
3. R_B and R_c are resistances, each is a two terminal element.
Components are interconnected by interconnecting component terminals. The interconnected terminal is called a **Node**. There is also a ground terminal G.

All components are physical devices. However, some physical devices can be modeled by other devices. Transistor can be modeled as such.

Physical devices are modeled by some other circuit elements



Transistor is modeled by a Dependent Current Source controlled by a voltage and two other resistances.

Principles of Electrical Engineering I

Fundamental System of Units

Table 1-1: Fundamental SI units.

Dimension	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Electric Charge	coulomb	C
Temperature	kelvin	K
Amount of substance	mole	mol

Some people consider Electric Current as the fundamental unit, and some others consider Electric Charge as the fundamental unit.
Electric charge on an electron, denoted by e, is said to be negative and has a value of 1.6 (10^{-19}) coulombs. Charge is quantized.
e is the smallest possible charge. Hence **any charge is an integral multiple of e.**

Money is quantized; in US cent is the smallest unit of currency.

Physical separation of charges generates an electric field and electric potential, just like physical separation of masses generates a gravitational field and gravitational potential.

Voltage is the result of separation of charges.

We will expand on this

Movement or flow of charges generates electric current, just like flow of water generates water current.

Current is the result of flow of charges.

TABLE 1.3 Standardized Prefixes to Signify Powers of 10

Prefix	Symbol	Power
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
deka	da	10
hecto	h	10^{2}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

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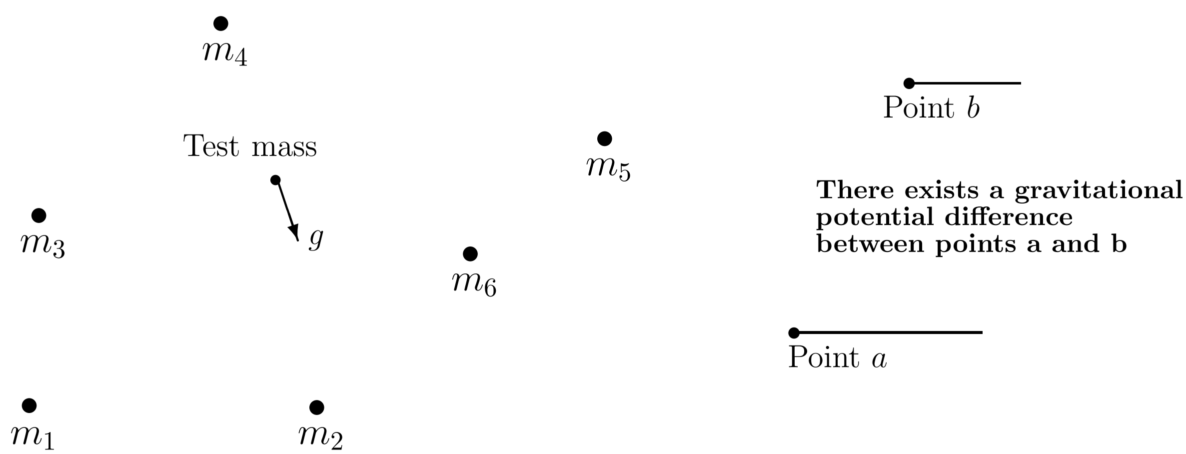
Gravitational field, intensity, potential difference

In physics or elsewhere, we learned the following concepts:

Gravitational Field: Masses attract each other with a force given by Coulomb's law. Such a force is conceptually thought of being the result of a gravitational field. That is, masses separated in space give rise to the concept of gravitational field. We say the gravitational field generated by separation of masses exerts a force on a test mass. The force exerted on a test mass of unit value is called **Gravitational Intensity** or **Gravitational force**, often denoted by g.

Gravitational Potential difference: The gravitational potential difference between two points 'a' and 'b' is simply the work required to be done in moving a unit mass from one point to another. To be precise, the work required to be done by an external agent in moving a **unit mass** from point 'a' to point 'b' is the gravitational potential rise of 'b' over 'a'; if point 'a' is considered as the **reference** or **ground** point, then the work required to be done is simply called the gravitational potential of point 'b'.



If g is a constant, the gravitational potential of a point is in fact can be represented by its height over a reference point.

Water (mass) flowing down the gravitational potential hill loses its potential.

Water (mass) can be pumped up the gravitational potential hill, in so doing it gains its potential. Which is the agent doing the work? Pump.

Page numbers are not always reliable as some other pages are inserted into the text

Electric field, intensity, potential difference, voltage

There exists a perfect analogy between gravitational effects and electrical effects. Most of this page is just a duplicate of previous page; we simply replace the word 'mass' by the word 'charge', and similarly we replace the word 'gravitational' by 'electrical' or simply 'electric'.

Electric Field: Like charges repel each other with a force given by Coulomb's law. Such a force is conceptually thought of being the result of an electric field. That is, charges separated in space give rise to the concept of electric field. We say the electric field generated by separation of charges exerts a force on a test charge. The force exerted on a test charge of unit value is called **Electric Intensity** or **Electric force**, often denoted by $\mathcal{E}$.

Electric Potential difference: The Electric potential difference between two points 'a' and 'b' is simply the work required to be done in moving a **unit charge** from one point to another. To be precise, the work required to be done by an external agent in moving a unit charge from point 'a' to point 'b' is the electric potential **rise** of 'b' over 'a'; if point 'a' is considered as the **reference** or **ground** point, then the work required to be done is simply called the electric potential of point 'b'. Electric potential difference between two points 'a' and 'b' is traditionally called the **Voltage** across 'a' and 'b'; **Voltage** of point 'b' simply means the **rise** of electric potential of 'b' with respect to a reference or ground.

Votage of b with respect to a =

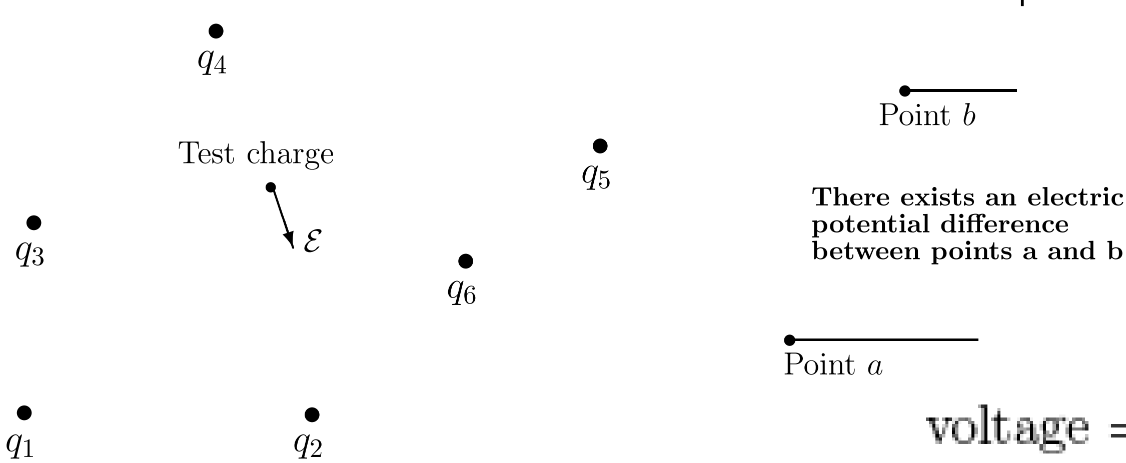
Work done by an agent in moving a **unit** charge from a to b

$$= \frac{dw}{dq}$$

where dw is work done in moving a charge of dq. If the locations of 'a' and 'b' is fixed, then voltage is simply the rate at which work is done with respect to charge.

Potential = Ability
Voltage is potential difference. It is simply `ability to do work'. Work is neither done nor absorbed until charge goes from one point to another.

Voltage is measured in volts;
Volt is abbreviated as V, and has the units of Joules per Coulomb.



$$\text{voltage} = \frac{dw}{dq} = v$$

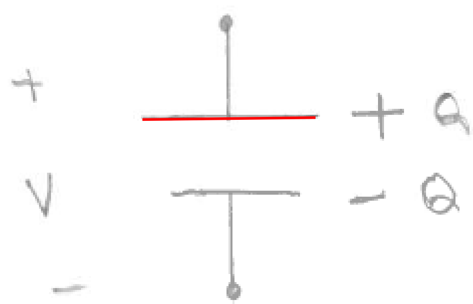
In analogy with the gravitational potential of a point, the voltage of a point can be thought of as its height over a reference point.

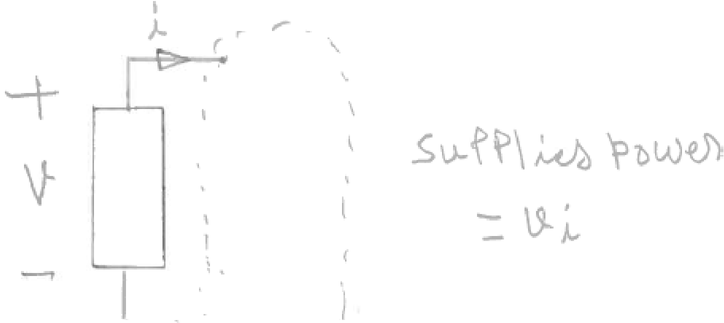
Charge flowing down the electrical potential hill loses its potential.

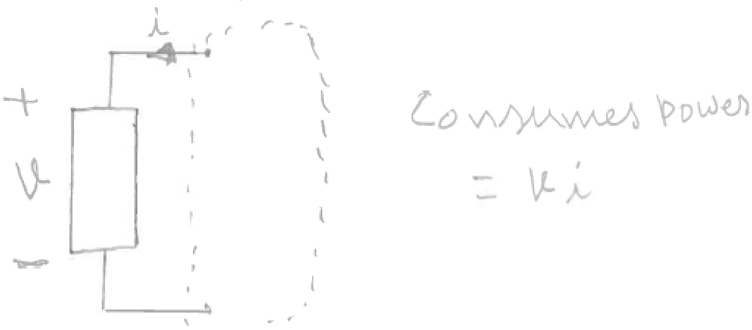
Charge can be pumped up the electrical potential hill, in so doing it gains its potential. Which is the agent doing the work ? The one that pumps the charge.

What is inside a battery?

It consists of two conducting plates immersed in a chemical compound or solution. When **charged**, positive charge collects on one plate and an equal negative charge collects on the other plate. This establishes a potential difference or voltage between the plates. When a positive current flows out of the positive terminal of the battery, battery is lifting positive charges from the negative plate towards the positive plate. In this way, it **supplies** power to components connected to the battery. If current flows into the positive plate, it **absorbs or consumes** power.





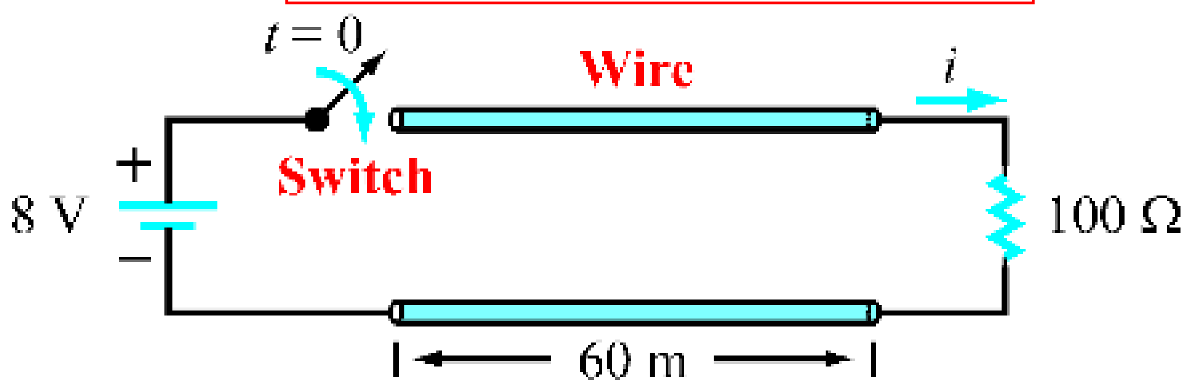


Battery

Physically, any **battery** consists of two plates in a medium of some chemical substrate. When charged properly, certain net amount of positive **charge** collects on one plate and an equivalent of negative charge on another plate. **Such a separation of charges establishes an electric field**, and consequently certain value of **potential difference or voltage** exists between the two plates. **The value of voltage is proportional to the value of charge**. If for some reason, the charge drifts away, the voltage reduces momentarily. However, if the battery is properly charged, the chemical medium restores the voltage at the prescribed level by pumping the charge from one plate to the other.

Comparison between a simple circuit containing a battery and a resistance on one hand, and the water pump, tubing, and water cooled radiator in an automobile on the other hand:

Analogy: Battery = Pump, Wire= Tubing, Resistance= Radiator



Suppose charge drifts away from positive plate via the upper wire shown in the circuit. Momentarily, the voltage between plates decreases. Then the chemical medium immediately pumps the charge from negative plate to the positive plate in order to maintain the voltage at the constant prescribed level. If the charge continuously drifts away from the positive plate via the upper wire, down the resistance, and then back to the negative plate via the lower wire, we then observe the battery continuously pumping charge from negative plate to the positive plate. This gives rise to continuous flow of charge or current. (The concept of current and resistance are explained shortly).

Think of the battery as the water pump, wiring as tubing, the resistance as the radiator in an automobile.

In the above circuit, which component does the work and which component absorbs the work?

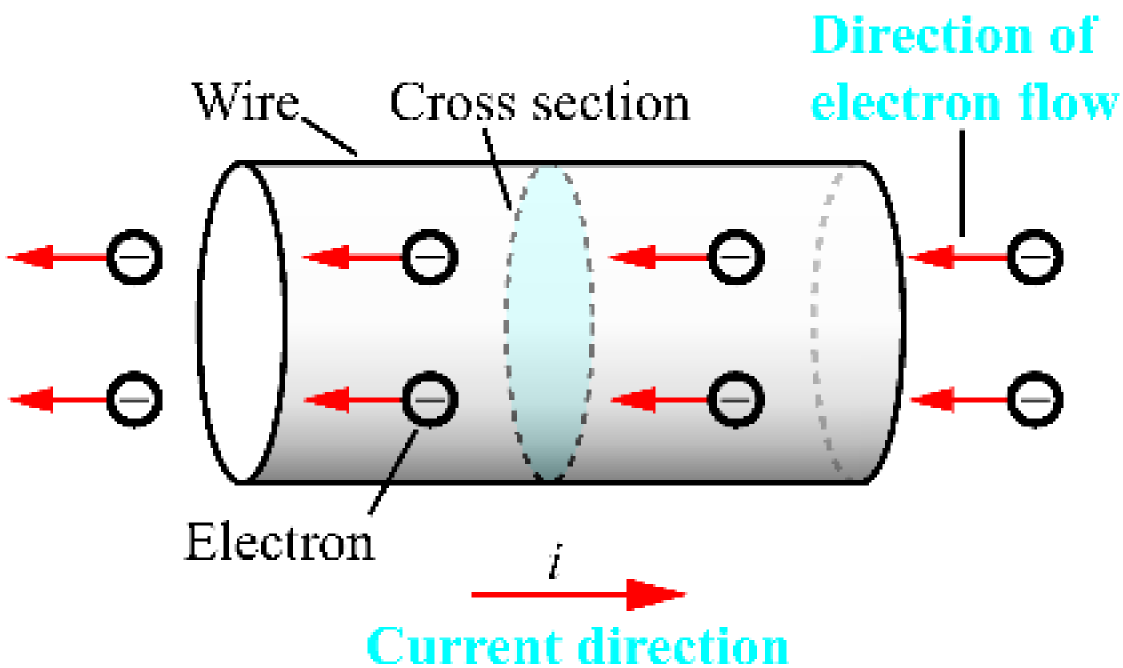
In an automobile cooling system, which component does the work and which component absorbs the work?

Concept of current: Electric current or simply current is the rate at which charge flows with respect to time through a cross section or boundary,

$$i = \frac{dq}{dt}$$

Current is denoted often by i, and is measured in Amperes.
Ampere or simply Amp is abbreviated as A, and has the units of coulombs per second.

The **direction of electric current** is defined to be the direction of flow that positive charges would flow, which is opposite to the direction of flow of electrons which carry negative charge.

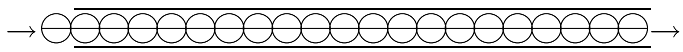


In general, both positive and negative charges flow through a given cross section. The current through the cross section is then the *net rate* of flow of *positive* charges. Consider a specific case of positively charged particles moving to the right and the negatively charged particles to the left. Then, the net effect of both actions is a positive charge moving to the right. The instantaneous current to the right is given by the equation

$$i = \frac{dq}{dt} = \frac{dq^+}{dt} + \frac{dq^-}{dt} \qquad \left.\begin{array}{l} q^+ \ \oplus\ \oplus\ \oplus\ \oplus\ \rightarrow \\ q^- \ \ominus\ \ominus\ \ominus\ \ominus\ \leftarrow \end{array}\right\} \text{implies that the net current is } \rightarrow$$

where q^+ and q^- indicate the <u>magnitudes</u> of positive and negative charges moving respectively to the right and left.

Before we cosider the mechanism of flow of electrons, let us consider movement of identical ping-pong balls in a pipe filled with such balls.

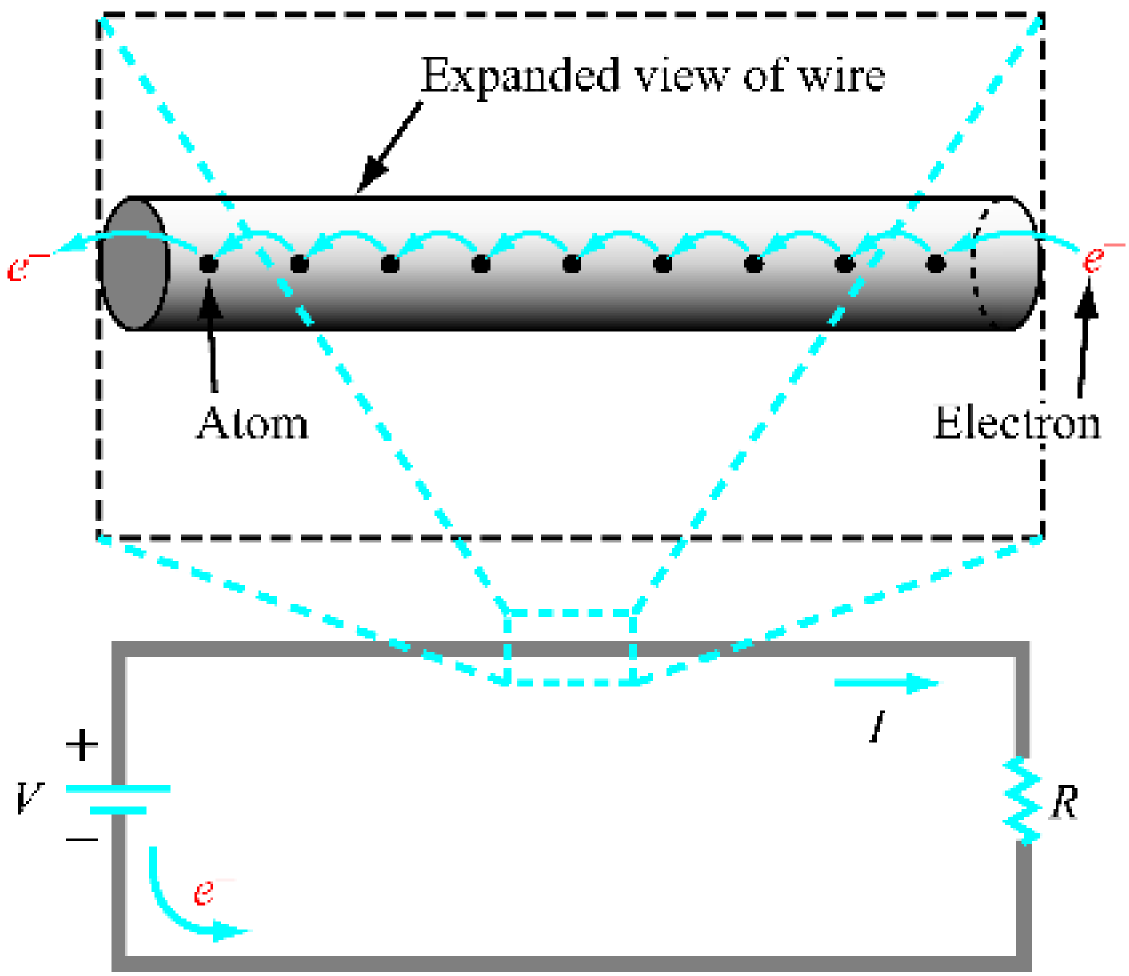


The diagram shows a pipe filled with ping-pong balls, each of which represented by a circle with its diameter. Let the diameter of ball be d and length of pipe be L. Let Δt be the time it takes to push a ball into the pipe on the left side. A ball comes out on the right side of pipe. What is the velocity of ball? We look at this in two ways:

$$\text{Actual drift velocity of ball} = \frac{d}{\Delta t}$$

However,

$$\text{Apparent observed velocity of ball} = \frac{L}{\Delta t}$$



Electron drift process: To explain this, consider the above figure. The conducting wire consists of atoms with loosely attached electrons called **valence electrons**. Electric force causes valence electrons jump from atom to atom. Let us explain this in detail. The positive charge on the positive terminal of the battery attracts near by valence electrons. This means the near by atoms which lost electrons are now positively charged (ionized), and hence attract valence electrons from their neighbors. This phenomena continues, and eventually ends up pulling electrons from the negative plate. The net result apparently **appears** to be this: The negative plate loses electrons rendering it to have less negative charge, and the positive plate gains electrons rendering it to have less positive charge. Also, it appears as if the same electrons lost by the negative plate are gained by the positive plate much like the flow of ping-pong balls in the pipe. We need to observe next that the battery in an attempt to restore its voltage (voltage is reduced if the charges on the plate are reduced) pumps back electrons gained by the positive plate into the negative plate via its chemical medium. Although electrons drift from atom to atom, it appears as if the electrons are circulating from negative plate to the lower wire, then to the resistance, then to the upper wire, then to the positive plate, then back to negative plate via the chemical medium. This gives rise to the notion of **conduction current**.

There are two velocities associated with the flow of charge much like the velocities associated with ping-pong balls in a pipe, the drift velocity of electrons or charges and the velocity of waves generated by the flow of charge. Drift velocity is much smaller than the velocity of waves which is in the order of light $c = 3 \times 10^8$ meters per second. The information travels at the velocity of waves. If a stone is thrown to skip on the surface of a fond, the water waves generated in the pond travel much faster than the stone travels. The scope of this course prohibits us to get into wave phenomena.

Power and Energy: Power is the rate at which work is done with respect to time, or rate at which energy is acquired or given away with respect to time. Mathematically, this can be expressed as

$$p = \frac{dw}{dt}, \tag{1}$$

where p is the power in watts (unit abbreviated as W), w is the work done or energy in joules, and t as usual is the time in seconds. Power is associated with moving charges or currents in an electric field. Indeed, by the chain-rule of calculus, we have

$$p = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = vi$$

$$p = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = vi$$

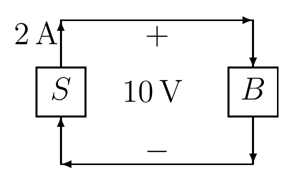
where voltage $= \frac{dw}{dq} = v$ in volts and current $\frac{dq}{dt} = i$ in amperes.

The above equation simply says that the power associated with a circuit element is simply the product of the current i through the element and the voltage v across it. One should be able to tell whether power is delivered by the element or absorbed by the element. By the definition of the voltage, whenever a current goes from the negative side of the voltage to the positive side of the voltage, the agent responsible for such an action delivers power.

Thus, in a circuit element, if a current i leaves the positive side of the voltage v, then the element delivers the power $p = vi$. Conversely, if a current i enters the positive side of the voltage v, then the element absorbs the power $p = vi$.

Consider the block diagram of a circuit on the right containing two elements S and B. The voltage across each element is 10 volts with the top terminal of each as a positive terminal. On the top side, a current of 2 amperes leaves the element S and enters the element B. On the other hand on the bottom side, a current of 2 amperes leaves the element B and enters the element S. Obviously, a charge of 2 coulombs per second (meaning 2 amperes of current) is pushed by the element S from the bottom side to the top side, *up the potential hill* of 10 volts. This charge of 2 coulombs per second flows through the element B from the top side to the bottom side, *down the potential hill* of 10 volts.

Anology: Water pump pushing or pumping fluid around the radiator.



We find that element S has to push 2 coulombs per second up the potential hill of 10 volts. Hence, by the definition of the voltage, element S is supplying (delivering) a power of $10 \times 2 = 20$ watts. In the case of element B, a charge of 2 coulombs per second is flowing down the potential hill of 10 volts. Hence, the element B is consuming (absorbing) a power of $10 \times 2 = 20$ watts. We observe that there is *conservation of power* in a circuit. That is, *whatever power is delivered by certain circuit elements is consumed by the other circuit elements.*

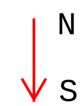
Reference direction and sign convention: Before we proceed further, we need to emphasize some fundamental but very elementary notations, reference directions, and sign conventions. This is the basic notation used throughout engineering, mathematics, and other scientific fields. For some students, this discussion is obvious, but others need to understand it critically. We illustrate here the reference directions and sign conventions by examples.

Example 1: Joe has a checking account. To keep track of the account, Joe defines the following equation:

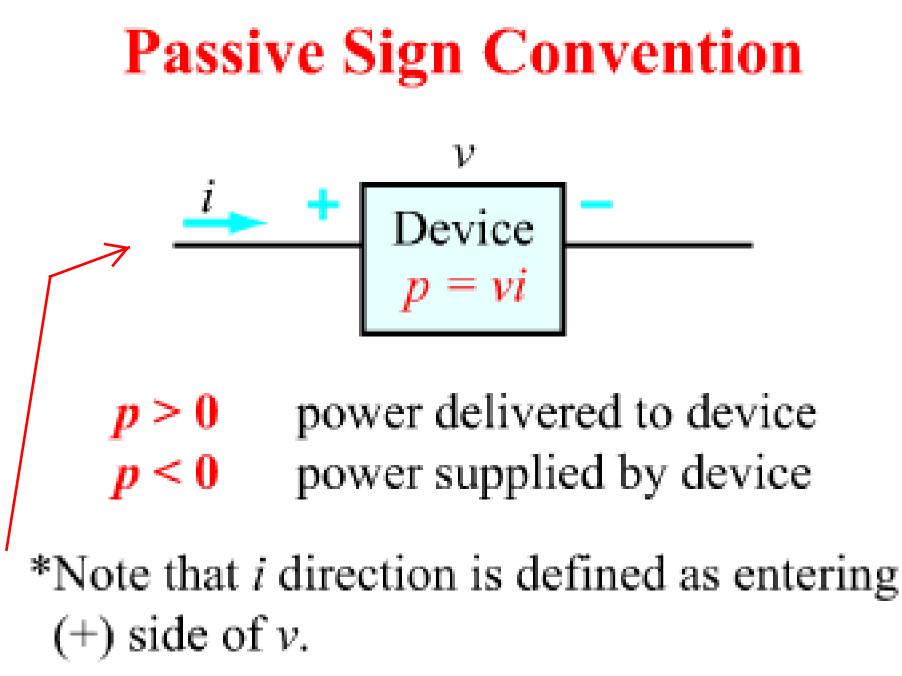
$$x = \text{Total deposits} - \text{Total withdrawals}$$

Be definition, if $x = 105$, bank owes Joe 105 dollars. On the other hand, if $x = -105$, bank owes Joe -105 dollars or equivalently Joe owes bank 105 dollars. Instead of always saying who owes whom, we let the sign of x tell the story of who owes whom. This mathematical sign convention is preferable when you set up equations and solve them. That is, we let the information of who owes whom be coded in the sign of x .

Example 2: We define below a sign convention for the direction of travel of an automobile. We say the automobile's speed s is positive if it is going from south to north. Thus, if $s = 30$ miles/hour, the automobile is indeed going with a speed of 30 miles/hour from south to north. On the other hand, if $s = -30$ miles/hour, the automobile is going with a speed of 30 miles/hour from north to south. Instead of always saying 'south to north' or 'north to south, we let the sign of s indicate the direction of travel.

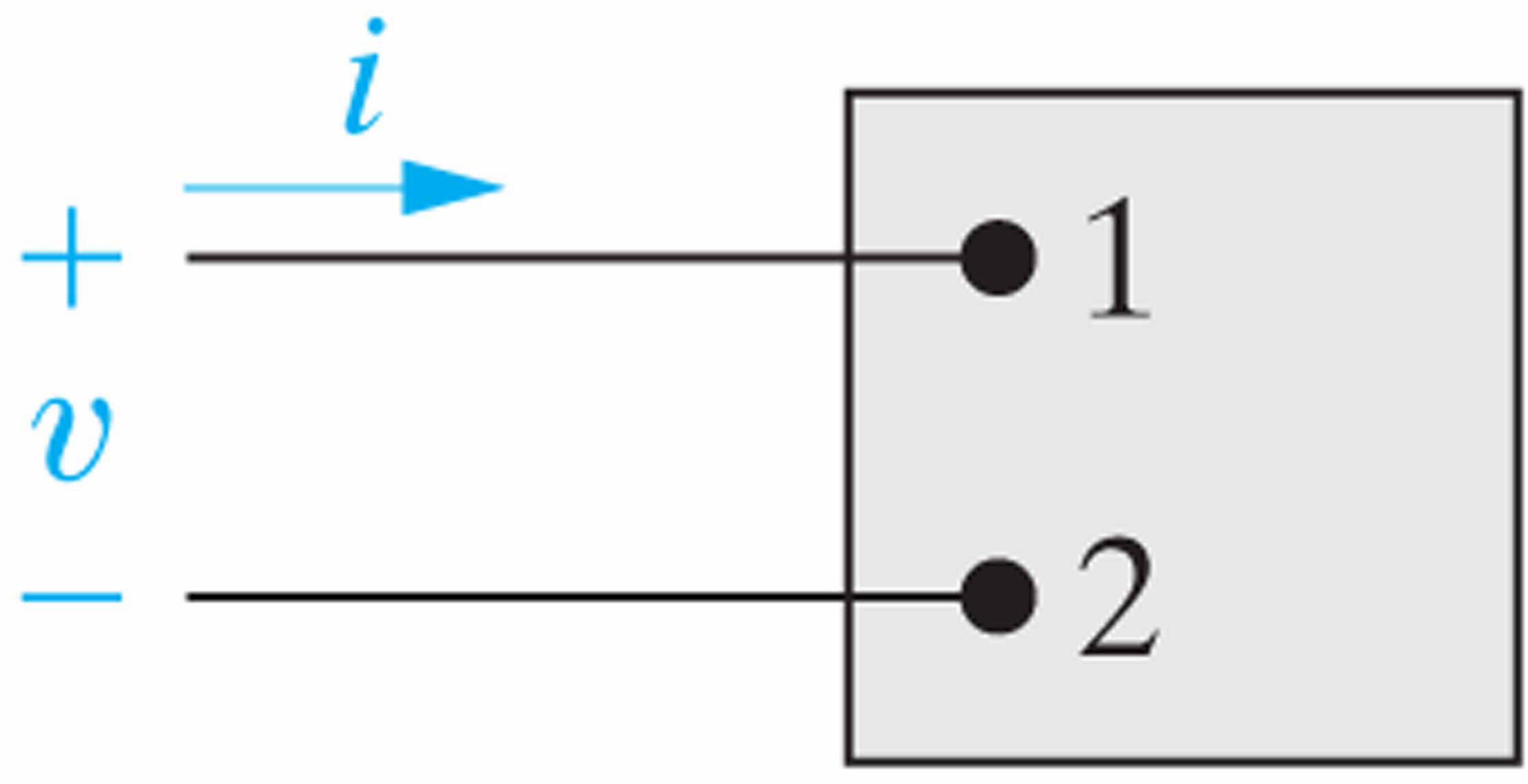


Example 3: In circuit analysis, we need to determine the voltage across a branch or current through a branch. To begin with, we do not know which side of a branch has a positive potential compared to the other side or which direction the current is going through it. In this case, we define variables along with their directions and then set up KCL and KVL equations. Once these equations are solved, we know the numerical values the chosen variables have. The sign of the numerical value of a voltage variable will inform us which side of it has a higher potential compared to the other side. Similarly, the sign of the numerical value of a current variable will inform us the direction of current through the branch.



Both voltage v and current i could have positive as well as negative values. As such, power can have both positive and negative values.

Figure 1.5 An ideal basic circuit element.



Carefully look at each branch
for voltage across it
and current through it
to determine the power either
supplied or consumed by it

Figure 1.6 Polarity references and the expression or power.

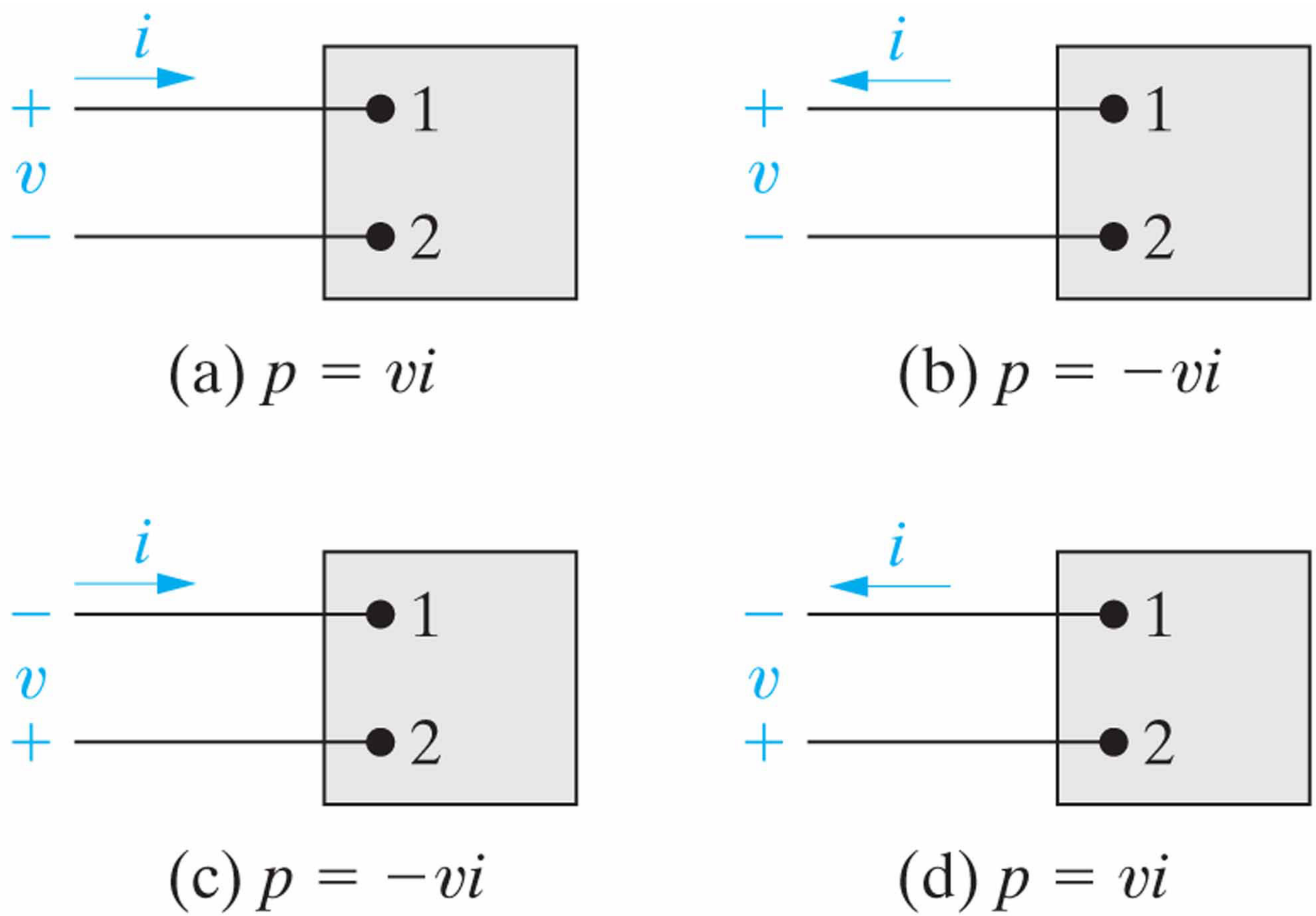


Figure P1.11

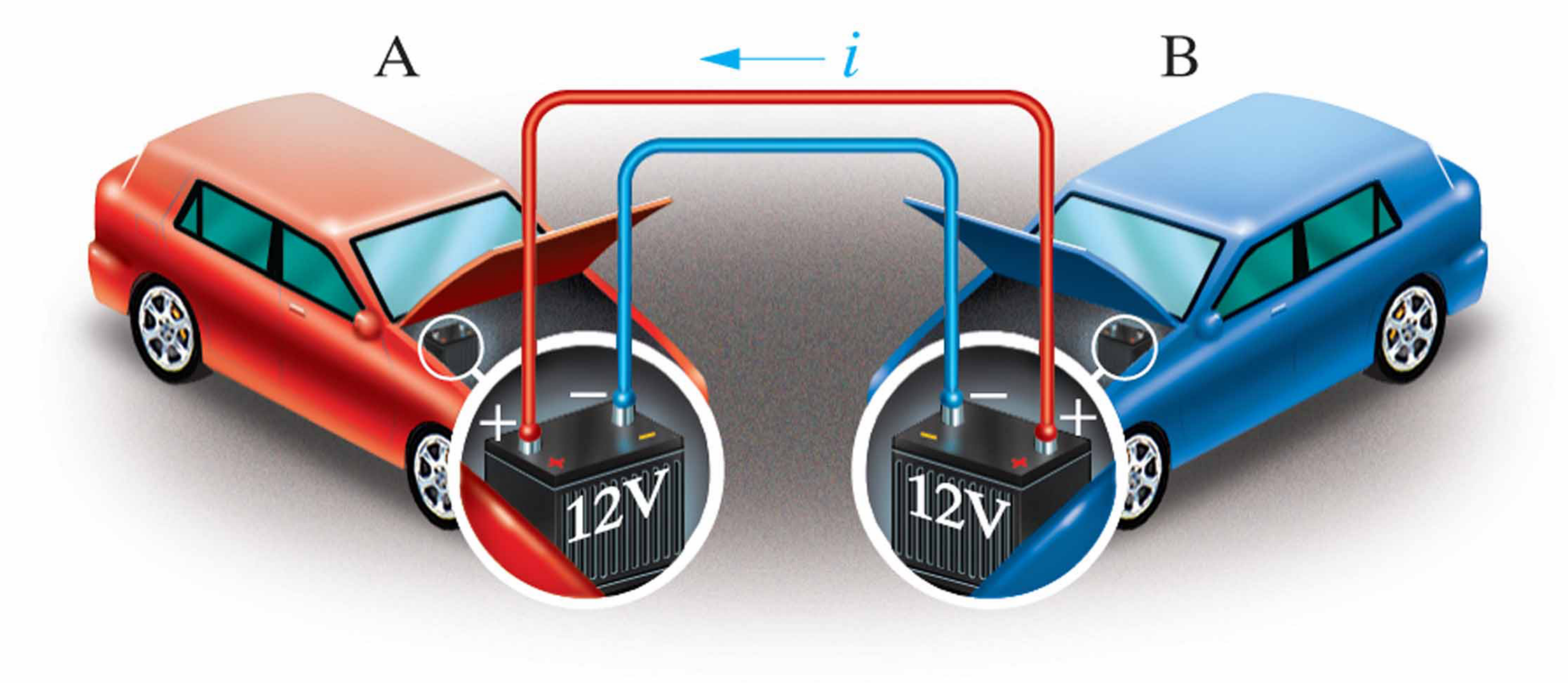


Figure P1.14

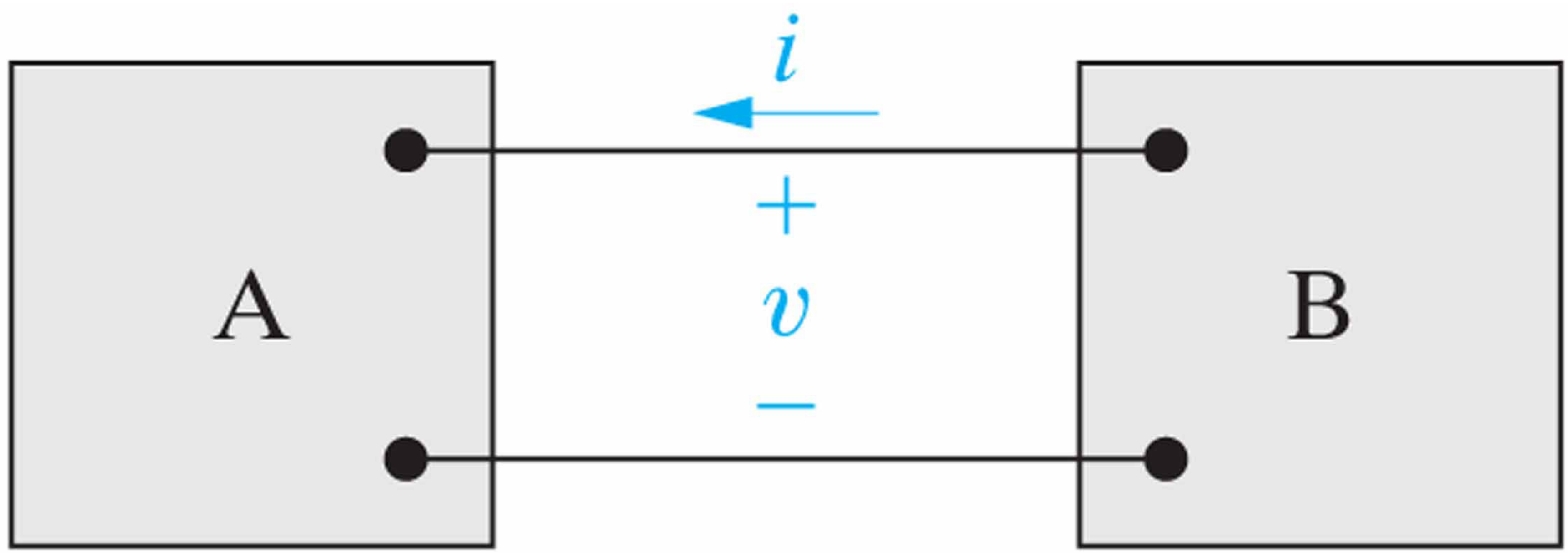
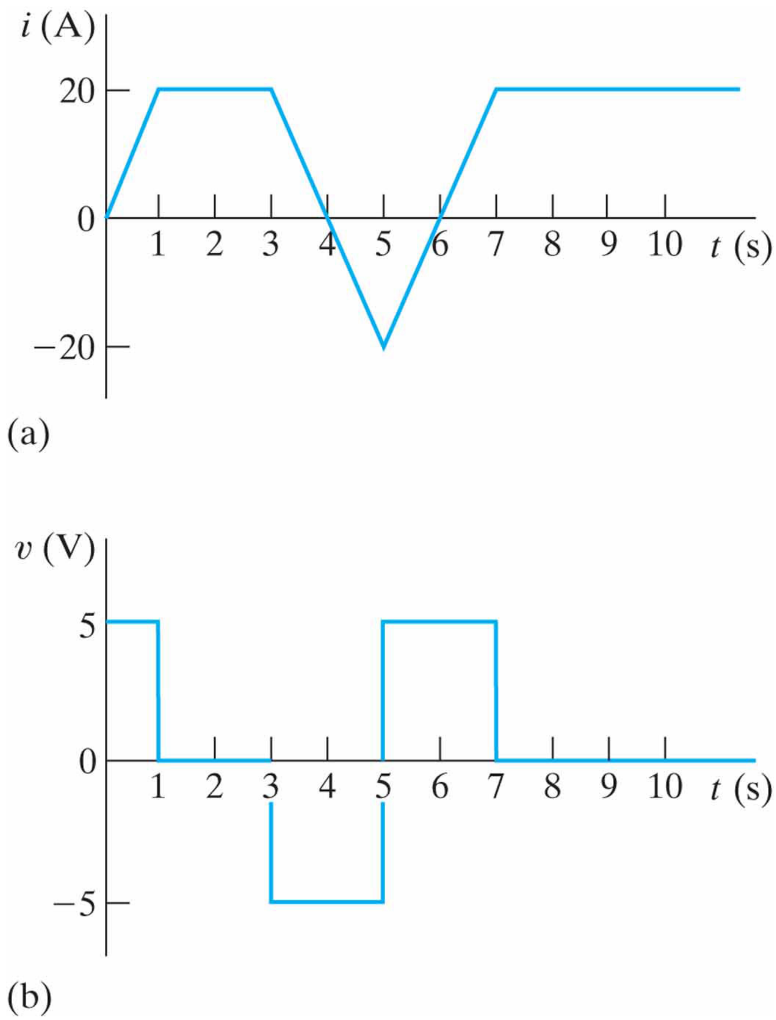
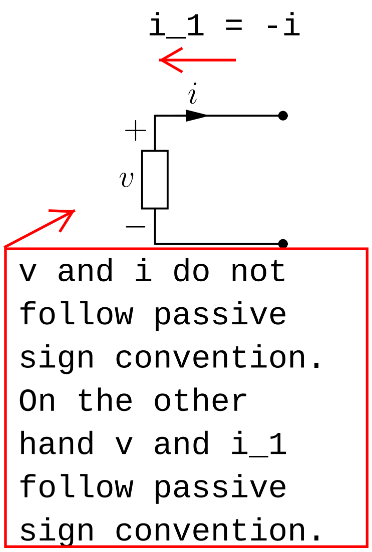


Figure P1.19

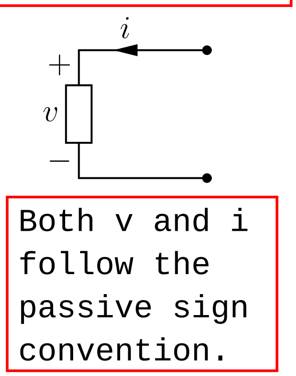


Example 4, Power computations and passive sign convention:

Power is generated or absorbed by a branch depending upon the directions (signs) of the voltage and current. Let us illustrate this.



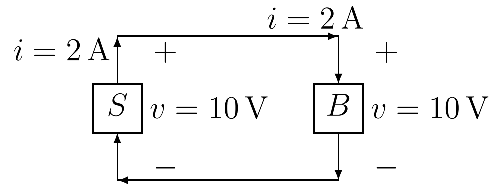
For the branch or element shown, the current i flows out of the positive side of voltage v.
For this case, the branch pumps the charge up the potential hill, and hence generates power.
Power **generated** (supplied) by the element $= vi$ watts.
We can restate the above as,
power consumed (absorbed) by the branch $= -vi$ watts.
Text book always calculates the power consumed by a branch. This is what the book calls as **Passive sign convention**. For the branch shown, by using passive sign convention, we say $p = -vi$ watts.



For the branch or element shown, the current i flows into the positive side of voltage v.
For this case, the charge goes down the potential hill and in so doing loses electric potential; and the released power is absorbed by the branch.
Power **consumed** (absorbed) by the branch$= vi$ watts.
We can restate the above as,
power generated (supplied) by the branch$= -vi$ watts.
Text book always calculates the power consumed by a branch
This is what the book calls as **Passive sign convention**, thus in such a passive sign convention $p = vi$ watts.

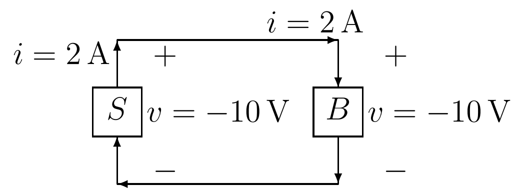
As mentioned above, one must adhere to reference directions and follow accordingly the sign convention. In other words, any scientific analysis uses variables for the concerned quantities with some chosen reference directions. After the analysis, if a certain variable turns out to be negative, then one can interpret this as the corresponding quantity having a magnitude equal to the absolute value of the variable and a direction opposite to the assumed one. The following situations are encountered in connection with power calculations. In all the following four illustrations, **for the indicated reference directions for the voltage as well as current, the power $p = vi$ is consumed by the element B, or the power $p = vi$ is generated by the source S.** We will concentrate our discussion on element B, you could analyze in a similar way what the source S is doing.

In this case, since $v > 0$ and $i > 0$, the power $p = vi$ consumed by the element B turns out to be positive and is given by $p = vi = 20$ W. Indeed, the element B is consuming a power equal to $vi = 20$ W.



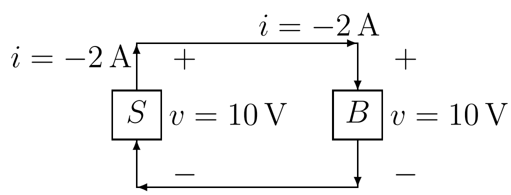
Element S is generating a power of 20 watts.

In this case, since $v < 0$ and $i > 0$, the power $p = vi$ consumed by the element B turns out to be negative and is given by $p = vi = -20$ W. Actually, the element B is generating a power equal to $|vi| = 20$ W.



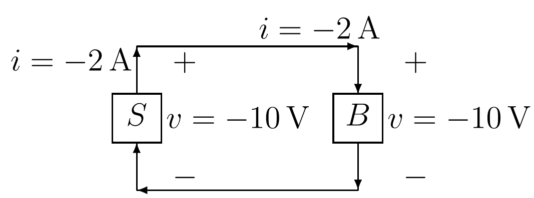
Element S is consuming a power of 20 watts

In this case, since $v > 0$ and $i < 0$, the power $p = vi$ consumed by the element B turns out to be negative and is given by $p = vi = -20$ W. Actually, the element B is generating a power equal to $|vi| = 20$ W.



Element S is consuming a power of 20 watts

In this case, since $v < 0$ and $i < 0$, the power $p = vi$ consumed by the element B is positive and is given by $p = vi = 20$ W. Thus, indeed, the element B is consuming a power equal to $vi = 20$ W.



Element S is generating a power of 20 watts

Discussion of some circuit notations:

Ground

Voltmeter and Ammeter

Open Circuit

Short Circuit

Single Pole Single Throw Switch (SPST)

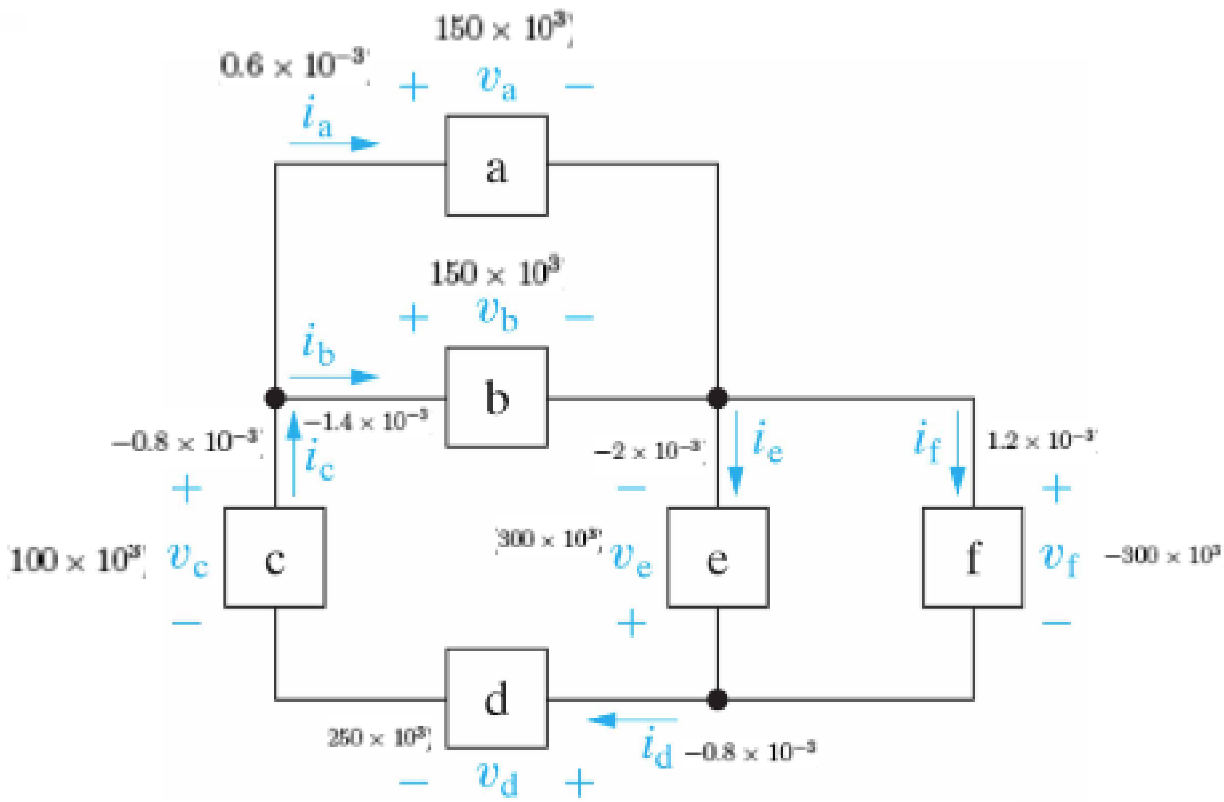
Single Pole Double Throw Switch (SPWT)

Problem:

Let us next consider a real circuit.

$$p_a = v_a i_a = (150 \times 10^3)(0.6 \times 10^{-3}) = 90 \text{ W}$$

$$p_b = v_b i_b = (150 \times 10^3)(-1.4 \times 10^{-3}) = -210 \text{ W}$$



$$p_a = v_a i_a = (150 \times 10^3)(0.6 \times 10^{-3}) = 90 \text{ W}$$

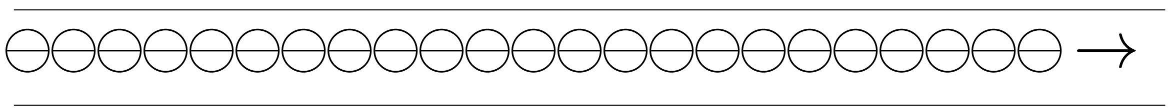
$$p_b = v_b i_b = (150 \times 10^3)(-1.4 \times 10^{-3}) = -210 \text{ W}$$

$$p_c = -v_c i_c = -(100 \times 10^3)(-0.8 \times 10^{-3}) = 80 \text{ W}$$

$$p_d = v_d i_d = (250 \times 10^3)(-0.8 \times 10^{-3}) = -200 \text{ W}$$

$$p_e = -v_e i_e = -(300 \times 10^3)(-2 \times 10^{-3}) = 600 \text{ W}$$

$$p_f = v_f i_f = (-300 \times 10^3)(1.2 \times 10^{-3}) = -360 \text{ W}$$



Problem: Consider a conductor. Let
Cross sectional area $= A$ meter^2
electrons density $= \rho$
(number of free electrons per cubic meter is ρ).

Find the current i in the conductor due to the electrons flowing with an
average velocity $= u$ meters/sec (Direction →)
perpendicular to the area of cross section.

Consider the figure. If we could collect all the electrons passing in one second through the point 'a', we could have collected all the electrons that are in u meters of the conductor. The current flow equals the charge on all the electrons in u meters of the conductor.

volume $= A\,u$
Number of electrons$= \rho\,A\,u$
Charge on an electron $= -1.6022 \times 10^{-19}$
Total charge flowing per second $= i$. Hence
$i = 1.6022 \times 10^{-19}\,\rho\,A\,u$ (← Direction)

If $A = 4 \times 10^{-12}$ m^2, $\rho = 10^{29}$, $u = 156.05$ m/s, find the current i. **(**$i = 10$ **Amperes)**

RUTGERS
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332:221 Principles of Electrical Engineering I

1. First Home-work of this week is due on Thursday, Sept 9, 2011
2. Second Home-work of this week is due on

Why not

No Kidding, all HW must be done by yourself

3. **Print the enclosed HW and do all your work right on the printed sheets in the space provided for each aspect of the problem. Otherwise, it is not graded.**
4. Write your name in legible CAPITAL LETTERS, otherwise HW is not graded.
5. Staple all the sheets that pertain to each HW problem separately. Each HW problem is graded separately, so do not staple different problems together.
6. **Submit the HW at the beginning of class,** HW is not accepted at the end of class. Late submission has the same effect as no submission.
7. To get credit, your submitted work must be self contained and easy to follow. All the directions of variables in a circuit must be marked, i.e. you must draw appropriate circuit or circuits and mark the variables and their directions.
8. Keep a copy of whatever you submit. Some of your HW may not be returned. Some of the unreturned HW will be looked at the time of giving the final grade especially when you are on the border of two letter grades.
9. **Solutions of these HW problems will not be posted. However, you are encouraged to come to office hours to discuss any aspects related to these problems as well as any aspects of the course.** Email correspondence about HW must be brief. It is time consuming to write lengthy email responses. Note that there are over 120 students enrolled in the course.
10. Like in the case of driving an automobile, laws of Circuit Analysis are simple and straightforward. However, **it takes practice to be at home with Circuit Analysis. This is where doing HW plays a significant role.**

332:221 Principles of Electrical Engineering I

Average Value and RMS value of a signal

This home-work problem is based on your knowledge of calculus. It is collected and graded.

Consider a time varying signal $x(t)$. Then, the average of the signal $x(t)$ over the period T is defined as,

$$\text{Average of } x(t) \text{ over the period } T = \frac{1}{T}\int_0^T x(t)dt.$$

The Root Mean Square value or simply called RMS value of $x(t)$ is defined as,

$$\text{Effective value of } x(t) \;=\; \underset{\text{Root}\nearrow}{\sqrt{\underbrace{\frac{1}{T}\int_0^T}_{\text{Mean}} \underbrace{x^2(t)}_{\text{Square}}\, dt}} = \frac{1}{T}\int_0^T x^2(t)dt..$$

Problem 1a: Determination of the Average and RMS value of a sinusoidal signal: Let us consider a sinusoidal signal

$$x(t) = A\cos(\omega t + \theta),$$

where A is the Amplitude, ω is the Angular frequency, and θ is the phase angle. Note that $\omega = \frac{2\pi}{T}$ where T is called the Period.
The Average value of $x(t)$ is given by

$$\frac{1}{T}\int_0^T x(t)dt = \frac{1}{T}\int_0^T A\cos(\omega t + \theta)dt$$

space
to
complete
integration

$= 0$ (The average is zero).

Use trigonometric identity,
$2\cos^2(\alpha) \;=\; 1 + \cos(2\alpha)$.

The RMS value of $x(t)$ is given by

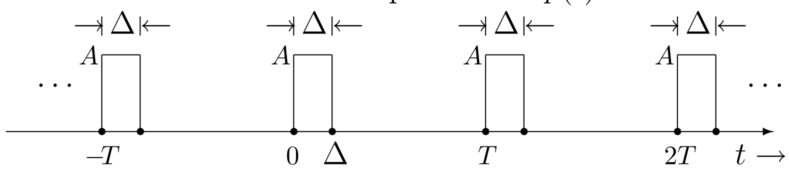
$$\sqrt{\frac{1}{T}\int_0^T x^2(t)dt} = \sqrt{\frac{1}{T}\int_0^T A^2\cos^2(\omega t + \theta)\, dt} = \sqrt{\frac{A^2}{2T}\int_0^T [1 + \cos(2\omega t + 2\theta)]\, dt}$$

space
to
complete
integration

$$= \frac{A}{\sqrt{2}} \text{ (The RMS value is Amplitude}/\sqrt{2}).$$

Problem 1b: Consider the periodic pulse train $p(t)$ shown below where the width of each pulse Δ equals $0.2T$. (The pulse train $p(t)$ is formed when a DC signal A is **ON** for a period Δ and **OFF** for a period $T - \Delta$ and so on. In this regard Δ is called the duty period.)

Periodic pulse train $p(t)$



Fill in the details

$$\text{The Average value of } p(t) \quad = \quad \frac{1}{T}\int_0^T p(t)dt =$$

$$= \quad \frac{A}{5}.$$

$$\text{The RMS value of } p(t) \quad = \quad \sqrt{\frac{1}{T}\int_0^T p^2(t)dt}$$

$$= \quad \frac{A}{\sqrt{5}}.$$