Performance of a Predetection EGC Receiver in Hoyt Fading Channels for Arbitrary Number of Branches

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***Abstract*—Simple and accurate expressions for the performance of an L branch equal gain combining receiver in independent Hoyt fading channels are presented. Using an approximate but highly accurate probability density function of the sum of Hoyt random variables, expressions for the average bit error rate for binary, coherent and noncoherent modulations have been obtained. The effect of the number of diversity branches on the normalized average output SNR is examined. The results have been verified against available results.**

***Index Terms*—Hoyt fading, Predetection EGC, ABER.**

I. INTRODUCTION

PERFORMANCE analysis of predetection equal gain combining (EGC) receivers in Hoyt fading channels is of interest for its possible application in mobile satellite communications. Although the average symbol error probability (ASEP) and other important performance measures for this receiver have been analyzed in [1] and [2], a mathematical expression for ASEP is not known for arbitrary number of fading branches. In a recent work an expression for ASEP has been obtained for an EGC receiver but it is limited to dual-diversity case only [3].

In this letter, using an approximate but highly accurate probability density function (PDF) of the sum of independent Hoyt random variables (RVs) [4], [5], we analyze the performance of a predetection EGC receiver in independent Hoyt fading channels for an arbitrary number of diversity branches. Mathematical expressions for the average output signal-to-noise ratio (SNR) and the average bit error rate (ABER) have been obtained for coherent, binary phase-shift-keying (BPSK), binary frequency-shift-keying (BFSK), differential coherent PSK (DPSK) and noncoherent FSK (NCFSK) modulation schemes.

II. CHANNEL AND RECEIVER

The channel has been assumed to be slow, frequency non-selective, with Hoyt fading statistics. The complex low-pass equivalent of the signal received at the l^{th} input branch $(1 \leq l \leq L)$ over one bit duration T_b can be expressed as,

$$r_l(t) = \alpha_l e^{j\phi_l} s(t) + n_l(t),\ 0 \leq t \leq T_b, \quad (1)$$

where $s(t)$ is the transmitted bit with energy E_b and $n_l(t)$ is the complex Gaussian noise having zero mean and two sided power spectral density $2N_0$, assumed identical for all the branches. The RV ϕ_l is the instantaneous phase and α_l is the Hoyt distributed instantaneous fading amplitude having PDF given by [6]

$$f(\alpha_l) = \frac{(1+q_l^2)\alpha_l}{q_l \Omega_l} e^{-\frac{(1+q_l^2)^2 \alpha_l^2}{4q_l^2\Omega_l}} I_0\left(\frac{(1-q_l^4)\alpha_l^2}{4q_l^2\Omega_l}\right),\ \alpha_l \geq 0, \quad (2)$$

where $\Omega_l = \mathrm{E}\left[\alpha_l^2\right]$, $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind and $0 \leq q_l \leq 1$ is the Hoyt fading parameter. The above density function can be expressed as a function of another Hoyt fading parameter $b_l \in [0,1]$ by substituting $q_l = \sqrt{\frac{1-b_l}{1+b_l}}$ [7]. In this letter, for the sake of convenience of presentation, we use the Hoyt PDF as a function of b_l and assume identical for all the branches i.e., $b_l = b, \forall\, l$.

The predetection EGC combiner cophases the signals received from different branches and produces the algebraic sum of these cophased signals at its output [7]. This is followed by a detector suitable to detect the signal corresponding to the modulation scheme used at the transmitter. The instantaneous output SNR γ of the predetection EGC receiver can be given by [7]

$$\gamma = (E_b/LN_0)\left(\alpha_1 + \alpha_2 + \ldots + \alpha_L\right)^2. \quad (3)$$

III. PERFORMANCE ANALYSIS

A. PDF of Output Signal-to-Noise Ratio

An expression for the PDF of γ in (3), $p_\gamma(\gamma)$ can be obtained if the PDF of $\sum_{l=1}^{L} \alpha_l$, which is the sum of L number of independent Hoyt RVs, is known. A useful and easy to handle expression for the PDF of the sum of arbitrary number of Hoyt distributed RVs has been presented in [4]. This is given as below:

Let α be the sum of L independent Hoyt distributed RVs $\alpha_l, l = 1, 2, \ldots, L$ i.e., $\alpha = \alpha_1 + \alpha_2 + \ldots + \alpha_L$. Then the PDF of α, $p_\alpha(\alpha)$ can be almost accurately given as

$$p_\alpha(\alpha) = \frac{4\sqrt{\pi}\mu^{\mu+\frac{1}{2}} h^\mu \alpha^{2\mu}}{\Gamma(\mu) H^{\mu-\frac{1}{2}} \Omega^{\mu+\frac{1}{2}}} e^{-\frac{2\mu h \alpha^2}{\Omega}} I_{\mu-\frac{1}{2}}\left(\frac{2\mu H \alpha^2}{\Omega}\right), \quad (4)$$

where $\Omega = \mathrm{E}\left[\alpha^2\right]$, $h \triangleq \left(2 + \eta^{-1} + \eta\right)/4$ and $H \triangleq \left(\eta^{-1} - \eta\right)/4$. The values of the parameters Ω, η and μ can be obtained by numerical evaluation of the expressions for moment based estimators given in [4, (4)-(8)].

The expression for $p_\gamma(\gamma)$ can be obtained by recognizing the relation between γ and α from (3) i.e., $\gamma = \frac{\alpha^2 E_b}{LN_0}$ and then applying the transformation of RV formula [8] to the PDF of

Manuscript received February 27, 2008. The associate editor coordinating the review of this letter and approving it for publication was J. van de Beek.

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Digital Object Identifier 10.1109/LCOMM.2008.080297.

α in (4). The obtained PDF after simplification can be given as

$$p_\gamma(\gamma) = \frac{2\sqrt{\pi}h^\mu}{\Gamma(\mu)} \left(\frac{\mu}{\bar{\gamma}}\right)^{\mu+\frac{1}{2}} \left(\frac{\gamma}{H}\right)^{\mu-\frac{1}{2}} e^{-\frac{2\mu h \gamma}{\bar{\gamma}}} I_{\mu-\frac{1}{2}}\left(\frac{2\mu H}{\bar{\gamma}}\gamma\right), \quad (5)$$

where $\bar{\gamma} = \frac{E_b}{LN_0}\Omega$.

B. Average Output Signal-to-Noise Ratio

The average output SNR $\bar{\gamma}$ of the EGC receiver can be obtained by averaging the PDF of γ in (5) i.e., $\bar{\gamma} = \frac{E_b}{LN_0}\Omega$. This can also be verified by substituting $\Omega = \mathrm{E}\left[\alpha^2\right]$ in (3). For an exponential power decay profile, the mean power of the lth branch is given by $\Omega_l = \Omega_1 \exp[-\delta(l-1)]$, where δ is the decay factor. Thus the average output SNR $\bar{\gamma}$ normalized with respect to the SNR of the first branch $\bar{\gamma}_1$, which we denote as $\bar{\gamma}_n$, can be given by

$$\bar{\gamma}_n = \frac{\bar{\gamma}}{\bar{\gamma}_1} = \frac{\Omega}{L\Omega_1} = \frac{\mathrm{E}[\alpha^2]}{L\Omega_1} = \frac{2f(L,\delta) + \pi\, {}_2F_1^2\left(\frac{1}{4}, -\frac{1}{4}; 1; b^2\right)\left[f^2\left(L, \frac{\delta}{2}\right) - f(L,\delta)\right]}{2L}, \quad (6)$$

where $f(l,\tau) \triangleq \frac{1-e^{-l\tau}}{1-e^{-\tau}}$ and ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gauss hypergeometric function. In the derivation of (6), we have used the relation $\alpha = \alpha_l + \alpha_2 + \ldots + \alpha_L$ and [4, (8)].

This is a general expression for $\bar{\gamma}_n$. For a uniform power decay profile (i.e., $\delta = 0$), it can be shown that (6) reduces to (11) of [1].

C. Average Bit Error Rate

In a digital communication system, for a given modulation scheme, ABER of the receiver can be obtained by averaging the conditional bit error rate (BER) $P(\epsilon|\gamma)$ over the PDF $p_\gamma(\gamma)$ of the receiver output instantaneous SNR γ. It can be given by

$$P_e = \int_0^\infty P(\epsilon|\gamma)\, p_\gamma(\gamma) d\gamma. \quad (7)$$

1) Coherent, BPSK and BFSK Modulations: For coherent, BPSK and BFSK modulations, the conditional BER can be given by $P_{coh}(\epsilon|\gamma) = \frac{1}{2}\mathrm{Erfc}\left(\sqrt{\gamma/g_c}\right)$, where $\mathrm{Erfc}(\cdot)$ is the Gaussian complementary error function and the parameter $g_c = 1, 2$ for BPSK and BFSK modulations, respectively [7].

An integral expression for ABER can be obtained by putting $P_{coh}(\epsilon|\gamma)$ and the PDF $p_\gamma(\gamma)$ from (5), into the R. H. S. of (7). The integral can be solved by substituting $\mathrm{Erfc}(x) = 1 - \frac{1}{\sqrt{\pi}}\gamma_f\left(\frac{1}{2}, x^2\right)$, where $\gamma_f(\cdot)$ is the incomplete gamma function, $I_{\mu-\frac{1}{2}}(\cdot)$ by [9, (8.445)] and then applying the identity [9, (6.445)] to the resulting integral. On simplifying the obtained expression, the ABER can be given as

$$P_{e,coh} = \frac{1}{2} - \frac{2\left(\sqrt{h}\mu g_c\right)^{2\mu}\sqrt{\bar{\gamma}}}{\Gamma(\mu)\left(\bar{\gamma} + 2\mu h g_c\right)^\beta} \sum_{k=0}^{\infty} \left(\frac{\mu H g_c}{\bar{\gamma} + 2\mu h g_c}\right)^{2k} \times \frac{\Gamma(\beta + 2k)\ {}_2F_1\left(1, \beta + 2k; \frac{3}{2}; \frac{\bar{\gamma}}{\bar{\gamma} + 2\mu h g_c}\right)}{k!\,\Gamma\left(\mu + k + \frac{1}{2}\right)}, \quad (8)$$

where $\beta = 2\mu + \frac{1}{2}$.

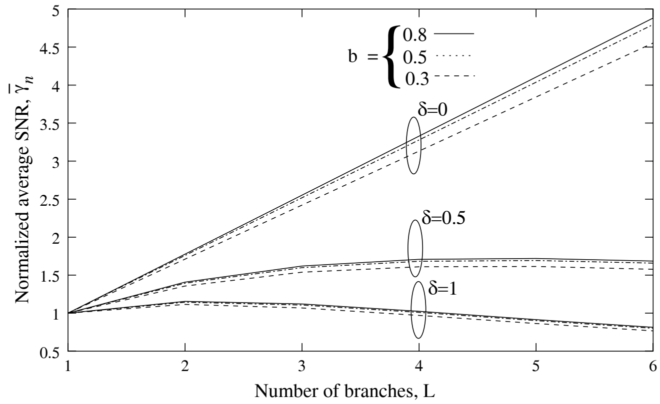

Fig. 1. Normalized average output SNR of the predetection EGC receiver in Hoyt fading channel.

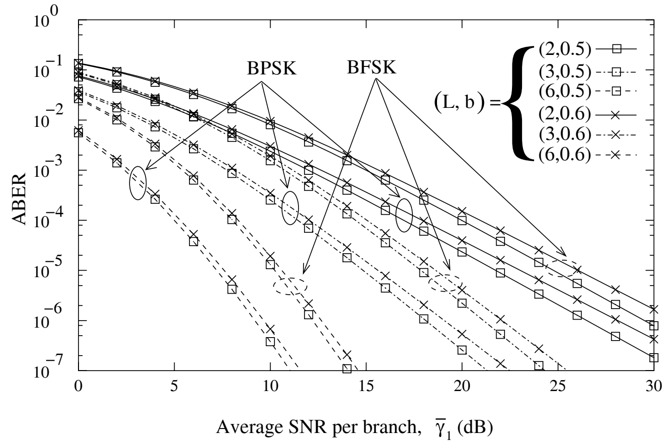

Fig. 2. ABER for predetection EGC receiver for BPSK and BFSK modulations in Hoyt fading channel.

2) DPSK and NCFSK Modulations: The conditional BER for DPSK and NCFSK modulations is given by $P_{ncoh}(\epsilon|\gamma) = \frac{1}{2}\exp(-\gamma/g_{nc})$, where the parameter $g_{nc} = 1, 2$ for DPSK and NCFSK modulations, respectively [7].

An integral expression for ABER can be obtained by putting $P_{ncoh}(\epsilon|\gamma)$ and the PDF $p_\gamma(\gamma)$ from (5), into the R. H. S. of (7). The integral can be solved by using [9, (3.361)]. The ABER expression after simplification can be obtained as

$$P_{e,ncoh} = \frac{1}{2}\left[\frac{2\sqrt{h}\mu g_{nc}}{\bar{\gamma} + 2\mu h g_{nc}}\right]^{2\mu} {}_2F_1\left(\mu, 1; 1; \left[\frac{2\mu H g_{nc}}{\bar{\gamma} + 2\mu h g_{nc}}\right]^2\right). \quad (9)$$

The Gauss hypergeometric function in (9) converges very fast as the magnitude of its argument inside the rectangular bracket is less than unity (since $h > H$).

IV. Numerical Results and Discussion

The normalized average output SNR $\bar{\gamma}_n$ obtained in (6) and the ABER expressions (8) and (9) have been evaluated numerically for different parameters of interest. In Fig. 1, $\bar{\gamma}_n$ vs. L has been plotted for different values of b and δ. It can be observed that the average output SNR is decreasing with increase in δ for any L, as expected. It can also be noted that for a given L and δ, the average SNR is increasing with increase in b. This result has been verified with the result presented in [1, Fig. 1] and is found to be closely matching.

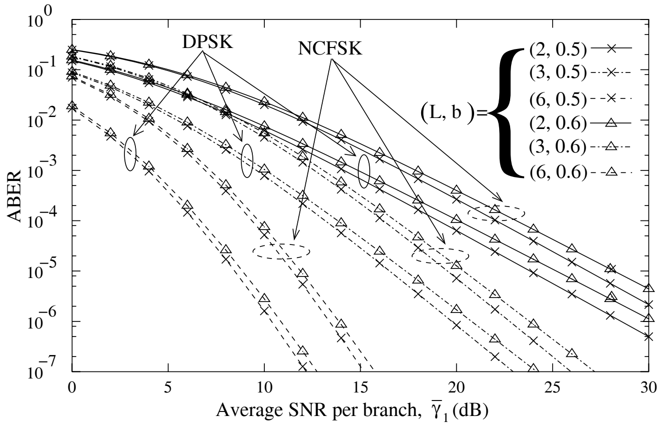

Fig. 3. ABER for predetection EGC receiver for DPSK and NCFSK modulations in Hoyt fading channel.

TABLE I
Ω, μ AND η PARAMETERS FOR THE PDF OF SUM OF HOYT RVs

L	$b = 0.5,\ \Omega_1 = 1$			$b = 0.6,\ \Omega_1 = 1$		
	Ω	μ	η	Ω	μ	η
2	3.5189	1.0610	0.2627	3.4939	0.9901	0.2551
3	7.5567	1.5899	0.2616	7.4818	1.4899	0.2530
4	13.1134	2.1187	0.2615	12.9635	1.9898	0.2524
5	20.1890	2.6475	0.2616	19.9392	2.4897	0.2522

TABLE II
NUMBER OF TERMS (n) REQUIRED FOR ACCURACY UP TO 7TH PLACE OF DECIMAL OF ABER IN (8)

SNR (dB)	$L = 2$		$L = 4$		$L = 5$	
$\bar{\gamma}_1$	$g_c = 1$	$g_c = 2$	$g_c = 1$	$g_c = 2$	$g_c = 1$	$g_c = 2$
0	15	14	17	17	18	19
10	15	15	18	18	18	21
20	15	14	16	17	17	17

In Fig. 2, ABER for BPSK and BFSK modulations have been plotted against average SNR of the first branch $\bar{\gamma}_1$ for $L = 2, 3$ and 6, each one for $b = 0.5$ and 0.6. In the similar manner ABER plots for DPSK and NCFSK are shown in Fig. 3. It can be observed from the curves given in Figs. 2 and 3 that the ABER is increasing with increase in b, which is expected, since increase in b implies increase in the severity of fading. It is important to observe the low SNR region ($\approx 0 - 10$ dB) of the plots shown in both the figures where the curves either crossover or overlap. It indicates that in this range of SNR, for a given L, the ABER of the receiver is almost independent of b and the modulation scheme used. This information is useful in the sense that it gives a flexibility in choosing a less complex modulation scheme for receivers likely to operate in low SNR environments. It can be verified that the ABER curve obtained for BPSK for $(L, b) = (2, 0.6)$ in Fig. 2 is matching with the the curve in [1, Fig. 4(b) for $M = 2$, $L = 2$ and $q = 0.5$] [1]. This verifies the correctness of our analytical results.

For numerical evaluation, the parameters Ω, μ and η of Hoyt distribution have been obtained, for each L, from [4, (4)-(8)]. For convenience and without loss of generality we have taken $\Omega_1 = 1$. Two sets of values have been tabulated in Table I for $b = 0.5$ and $b = 0.6$. In the numerical evaluation of (8), the involved infinite series has been truncated to its first 21 terms which is enough to achieve an accuracy in ABER at least at 7^{th} place of decimal digit. Table II shows some examples of the number of terms required to achieve this accuracy as a function of $\bar{\gamma}_1$, L and g_c. It can be observed that very few terms are required for the desired accuracy.

[1]applying the relation between b and q as mentioned in (2)

V. CONCLUSION

In this letter, we obtain simple and highly accurate expressions for ABER of a predetection EGC receiver with an arbitrary number of branches in independent Hoyt fading channels for binary, coherent and noncoherent modulation schemes. The direct approach of using the PDF of the sum of Hoyt RVs simplifies the final expressions and the evaluation complexity as well. Obtained numerical results for BPSK, BFSK, DPSK and NCFSK modulation schemes have been verified with the published results as applicable.

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